# Understanding Monty Hall Variants Through Analysis and Simulation 

## Switch or Stay?

- In front of you are three large haystacks
- One hides a needle
- Win a prize if you pick the one hiding the needle
- After you pick one, Monty Hall blows away another stack not containing needle
- You decline his offer to switch stacks
- Monty mostly blows away the other stack, leaving just a few straws of hay
- Should you take his next offer to switch?


Switch
Stay

## The Bayes' Theorem Solution

## Classic Monty

- Suppose player picks door 1, Monty opens door 3 , and player can switch to door 2
- Let $A$ be event that door 1 wins, $B$ be event that door 2 wins, and $C$ be event that Monty opens door 3
- Note: $P(C \mid B)=1$ since Monty must open door 3, and $P(C \mid A)=1 / 2$ since Monty has choice of opening door 2 or 3
- Bayes' theorem: probability that door 2 wins given Monty opens door 3 is

$$
\begin{aligned}
P(B \mid C) & =\frac{P(B) \cdot P(C \mid B)}{P(B) \cdot P(C \mid B)+P(A) \cdot P(C \mid A)} \\
& =\frac{\frac{1}{3} \cdot 1}{\frac{1}{3} \cdot 1+\frac{1}{3} \cdot \frac{1}{2}}=\frac{\mathbf{2}}{\mathbf{3}}
\end{aligned}
$$

## Forgetful Monty

- Use of Bayes' theorem same as Classic Monty
- However, $P(C \mid B)=1 / 2$ since Monty revealed goat randomly; thus



## 1000 Door Monty

- Redefine event $C$ as Monty opens doors 3-1000
- Note: $P(C \mid A)=\frac{1}{999}$ since Monty has 999 choices of door to leave closed; thus

$$
P(B \mid C)=\frac{\frac{1}{1000} \cdot 1}{\frac{1}{1000} \cdot 1+\frac{1}{1000} \cdot \frac{1}{999}}=\frac{\mathbf{9 9 9}}{\mathbf{1 0 0 0}}
$$

## Haystack Monty

- Redefine events $A$ as stack 1 is the winner, $B$ as stack 2 is the winner, and $C$ as Monty blows away stack 3
- Note: $P(C \mid B)=1$ since Monty must blow away stack 3 , and $P(C \mid A)=1 / 2$ since Monty can reduce stack 2 or 3
- Use of Bayes' theorem then identical to Classic Monty, so

$$
P(B \mid C)=\frac{\mathbf{2}}{\mathbf{3}}
$$

## Results

Version Probability of win if switching
Classic
1000 Door
$2 / 3$
$999 / 1000$
Forgetful
999/1000
Haystack

## The Monty Hall Problem

- Player presented with three doors
- One hides new car; others hide goats
- Player tries to win car by picking door

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- 
- Monty opens another door to reveal a goat
- Then offers player option to switch doors

Question: Should the player switch?
Answer: Yes; gives $2 / 3$ probability of winning

## Monty Hall Variants

- Forgetful Monty: After opening a door to reveal a goat, Monty says to himself, "Thank goodness, I forgot where the car was!"
1000 Door Monty: There are now 1000 doors. One hides the car, the others, goats. Monty opens 998 doors.
- Haystack Monty: There are three haystacks. Monty blows one away and the player declines to switch. He then reduces the other haystack to just a few straws of hay. Again, the player is given the chance to switch stacks.


## Simulation

- Simulation written in R models probability of winning by switching for each variant
- Function returns winning strategy for one trial, then repeated trials based on random sampling are run
- Data from repeated simulations show that fraction of games won by switching converges to actual probability of winning by switching

| Table 1: Classic |  | Table 2: Forgetful |  |
| :---: | :---: | :---: | :---: |
| Games played | Fraction won by switching | Games played | Fraction won by switching |
| 2 | 1 | 2 | 0.5 |
| 4 | 0.5 | 4 | 0.25 |
| 8 | 0.75 | 8 | 0.625 |
| 16 | 0.625 | 16 | 0.4375 |
| 32 | 0.75 | 32 | 0.53125 |
| 64 | 0.734375 | 64 | 0.5 |
| 128 | 0.6640625 | 128 | 0.507813 |
| 256 | 0.75 | 256 | 0.511719 |
| 512 | 0.685546875 | 512 | 0.498047 |



Figure 1: R plots show convergence to actual probability of winning by switching.

## Future Work

- Haystack Monty may provide more insight into probabilistic intuition
- Hypothesis: common assumption of greater probability that needle is hidden in larger, rather than reduced, haystack

| Table 3: 1000 Door |  | Table 4: Haystack |  |
| :---: | :---: | :---: | :---: |
| Games played | Fraction won by switching | Games played | Fraction won by switching |
| 2 | 1 | 2 | 1 |
| 4 | 1 | 4 | 0.25 |
| 8 | 1 | 8 | 0.625 |
| 16 | 1 | 16 | 0.75 |
| 32 | 1 | 32 | 0.625 |
| 64 | 1 | 64 | 0.671875 |
| 128 | 1 | 128 | 0.65625 |
| 256 | 1 | 256 | 0.648438 |
| 512 | 0.998046875 | 512 | 0.664063 |

haystack <- function() \{
need7e <- $\operatorname{sample}(1: 30000,1)$
winner <- ceiling(needle/10000)
player <- sample(1:3, 1)
monty <- sample(c(1:3) [-c(player, winner)], 1) reduced <- c(1:3)[-c(player, monty)]
if(winner==player) \{
reduced <- c(needle, -1)
strategy <- "Stay"
\}e1se\{
reduced <- c(-1, -2)
strategy <- "Switch"
\}
return(strategy)
\}
forgetful <- function() \{
scratch <- T
while (scratch==T) $\{$
car <- sample(1:3, 1)
player <- sample(1:3, 1)
monty <- sample(c(1:3) [-player], 1)
if(player==car)\{
scratch <- F
strategy <- "Stay"
\}else if(monty==car)\{
scratch <- T
strategy <- "Toss"
\}else\{
scratch <- F
strategy <- "Switch"
\} \# end if else
\} \# end while
return(strategy)
\} \# end func
Figure 2: Functions are written in R to model one trial of Haystack and Forgetful Monty.

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