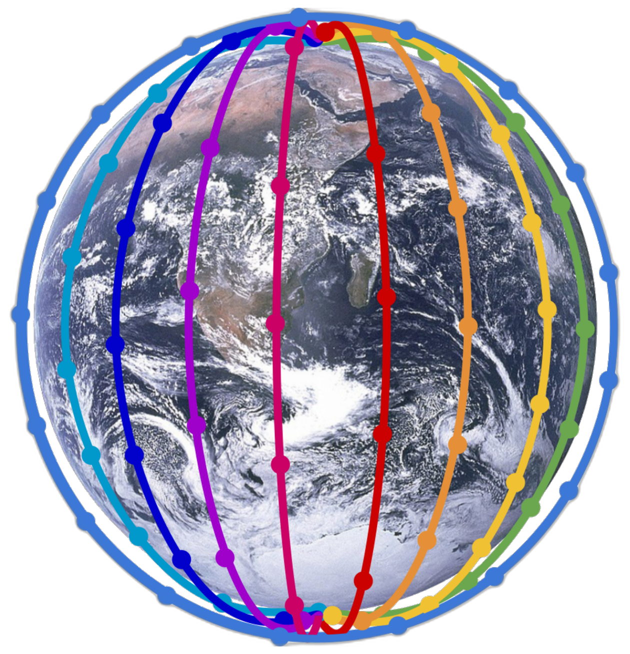
Distributed Kalman Filter Designs and Experiments for Proliferated Low Earth Orbit (pLEO) Satellite Constellations

Ashley Anderson [1] Sierra Dunn [2] Amit Harlev [3] Kaylee Rosendahl [4]

[1]Occidental College [2]Mount Holyoke College [3]Harvey Mudd College [4]Illinois Institute of Technology

Introduction

Proliferated low Earth orbit (pLEO) satellite constellations are comprised of hundreds to thousands of satellites operating at altitudes within 2000 km of the Earth's surface. Recently, pLEO satellite constellations have garnered interest due to their robustness, lower latency, and increased bandwidth compared to traditional satellite systems. However, the large number of satellites in pLEO constellations introduce new position, navigation, and timing (PNT) challenges. In particular, pLEO constellations cannot rely solely on information from ground stations for accurate PNT, making autonomous synchronization across the constellation a critical issue.

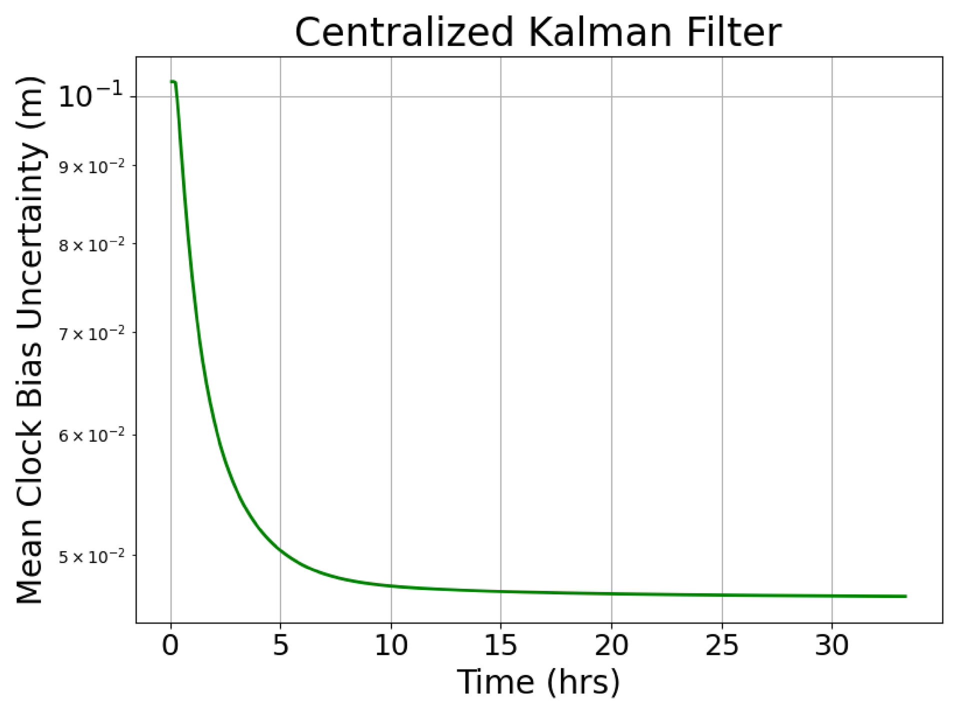


Project Goals

- Explore the use of distributed Kalman filters to autonomously synchronize satellite clocks across the constellation.
- Conduct numerical experiments comparing the centralized Kalman filter, a naive distributed filter, and a distributed approximation of the centralized Kalman filter known as the Decentralized Collaborative Localization (DCL) algorithm [1].

Centralized Kalman Filter

The centralized Kalman filter (CKF) is an optimal estimator that accounts exactly for the cross-covariances between different satellites' clocks over time. However, its implementation would require a prohibitive amount of communication between satellites, making it impractical as an actual solution. Despite this, it provides an ideal performance benchmark against which we can compare the performance of our distributed algorithms.



The Kalman Filter Equations

$$P_t^- = \varphi P_{t-1}^+ \varphi^T + Q$$

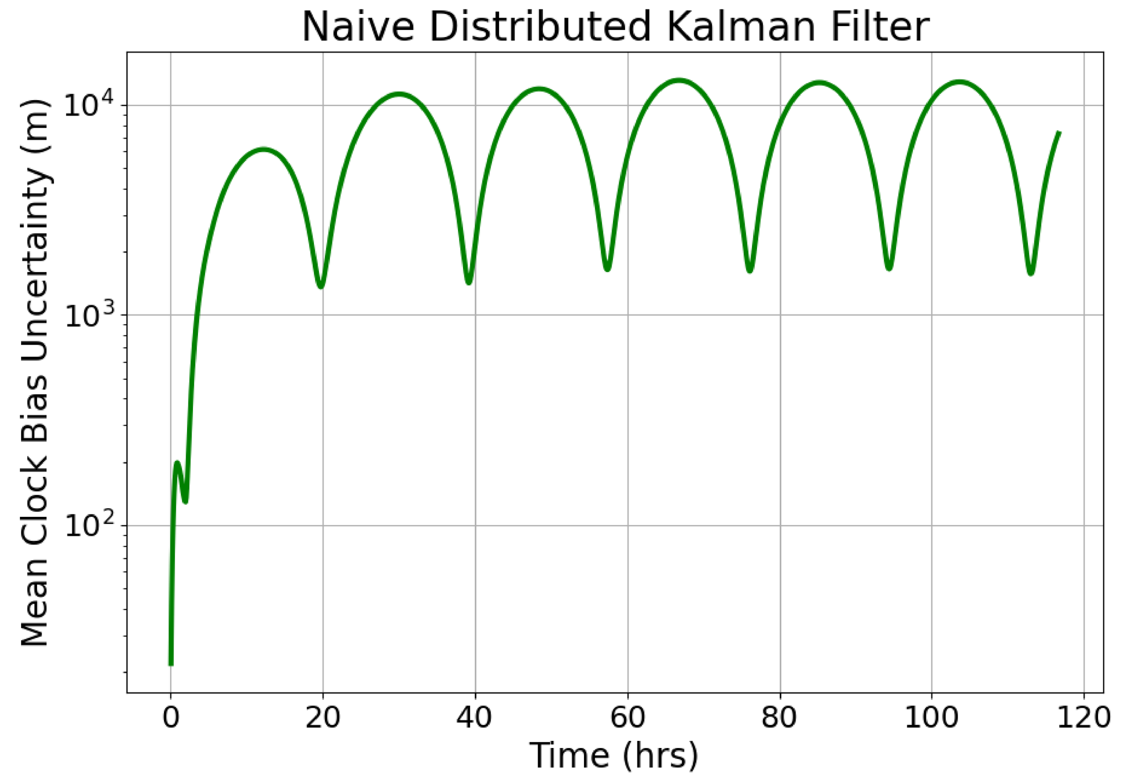
$$K_t = P_t^- H^T (H P_t^- H^T + R)^{-1}$$

$$P_t^+ = (I - K_t H) P_t^- (I - K_t H)^T + K_t R K_t^T$$

- P^- → a priori covariance matrix.
- P^+ → a posteriori covariance matrix.
- φ → system dynamics.
- K → Kalman gain.
- H → observation matrix.
- R → measurement noise.
- Q → process noise.

Naive Distributed Kalman Filter

The naive approach for addressing the communication requirements of the CKF involves running a separate Kalman filter for each satellite, rather than a single filter for the entire constellation. However, this approach ignores the cross-covariance terms that arise from measurements between satellites, leading to very high uncertainty.



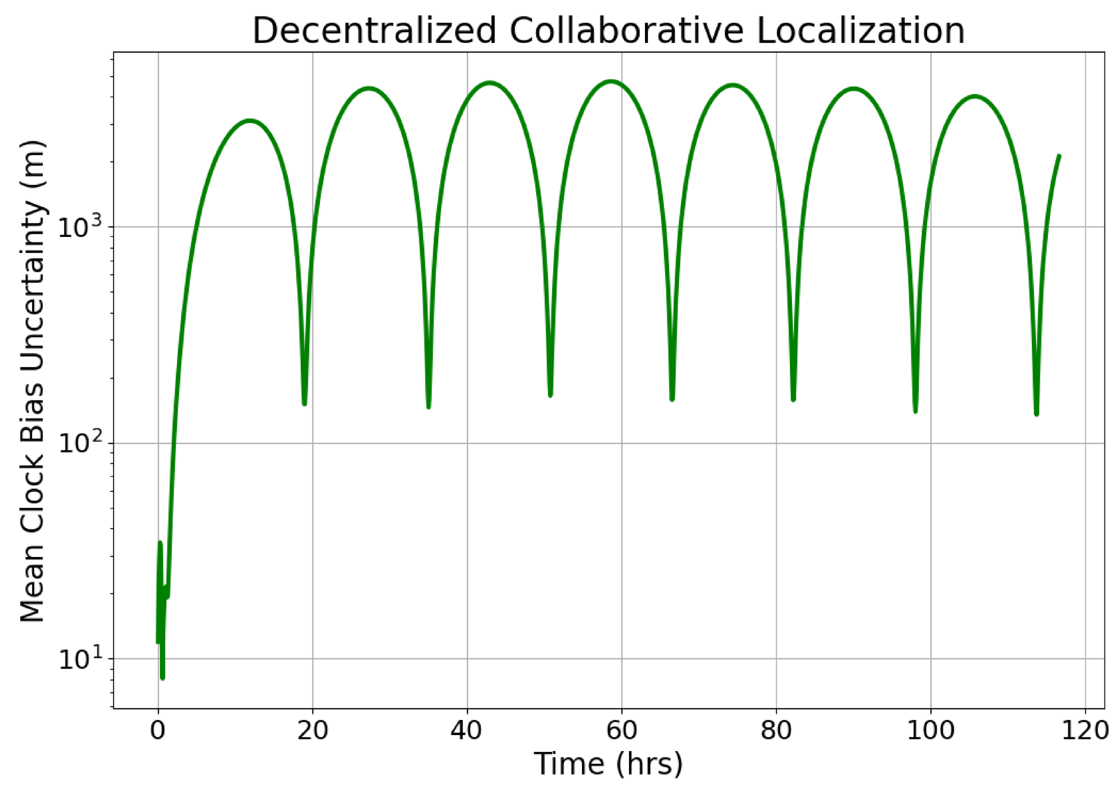
Decentralized Collaborative Localization

Decentralized collaborative localization (DCL) attempts to build on the naive distributed filter by approximating the relevant cross-covariances after any two satellites communicate [1]. After running the Kalman filter equations for a system of two satellites:

$$\text{covariance matrix } P = \begin{bmatrix} P_{i,t} & P_{i,j,t} \\ P_{j,i,t} & P_{j,t} \end{bmatrix} \text{ for satellites } i \text{ and } j,$$

DCL approximates the new covariance of the two satellites with every other satellite in the constellation:

$$\text{for every other satellite } k \rightarrow \begin{cases} P^+_{i,k,t} = P^+_{i,t}(P^-_{i,t})^{-1}P^-_{i,k,t} \\ P^+_{j,k,t} = P^+_{j,t}(P^-_{j,t})^{-1}P^-_{j,k,t} \end{cases}$$



Acknowledgements

This project was jointly supported by the Aerospace Corporation and the Institute for Pure and Applied Mathematics at UCLA.

Optimizing DCL

Tuning the Filter

We vary the process noise and initial uncertainty by scaling them to find the best performance for our system.

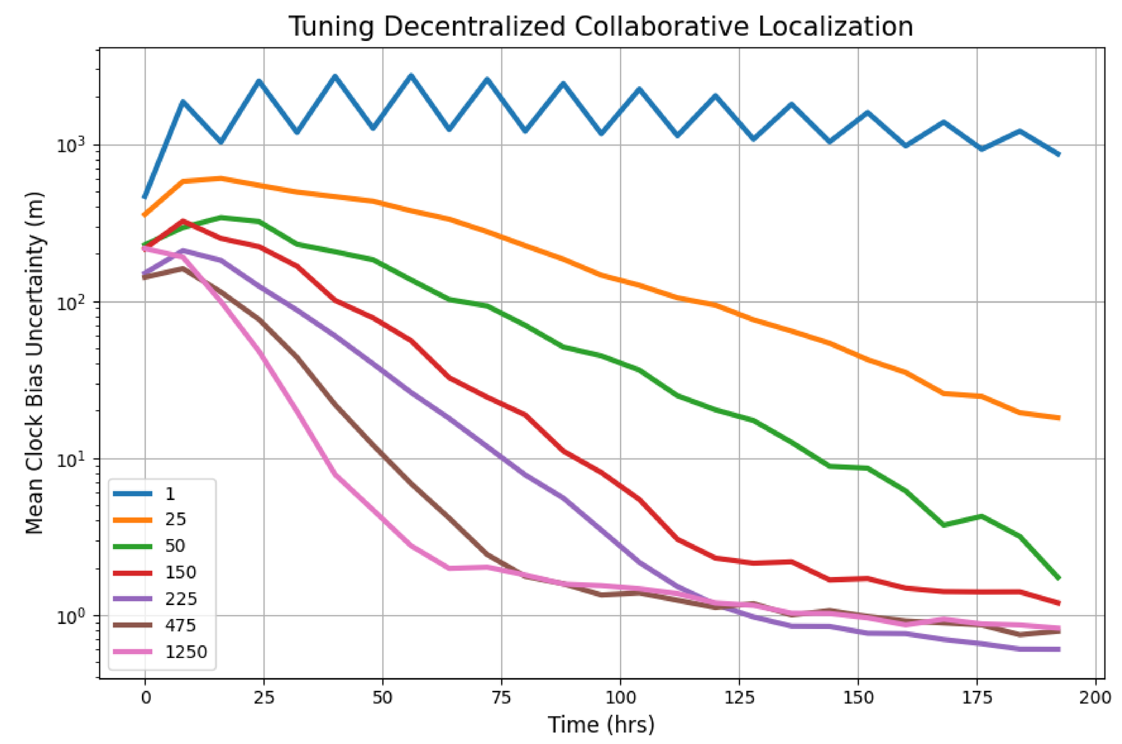


Figure 1. Performance of DCL when process noise is scaled by different values.

Satellite Communication Order

By fixing the order of satellite communications to propagate out from a single point, we can ensure that information travels further across the constellation each time step.

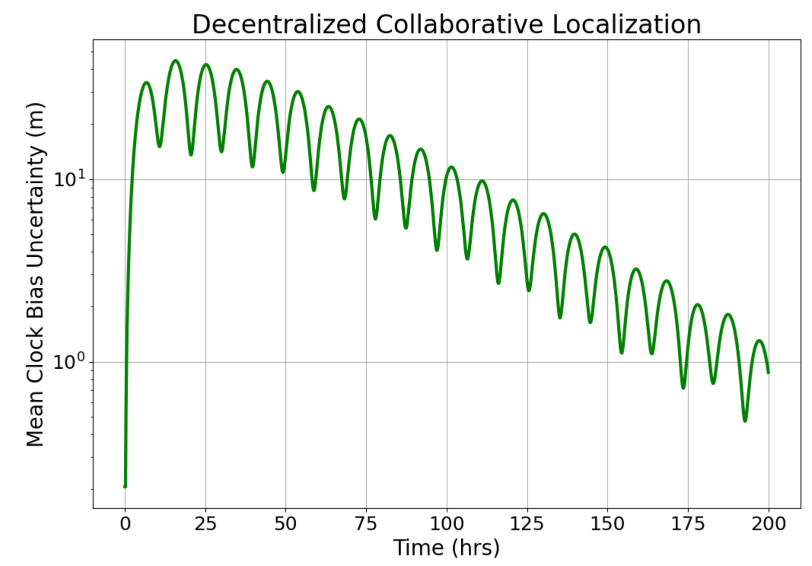


Figure 2. DCL performance using only optimized communication order.

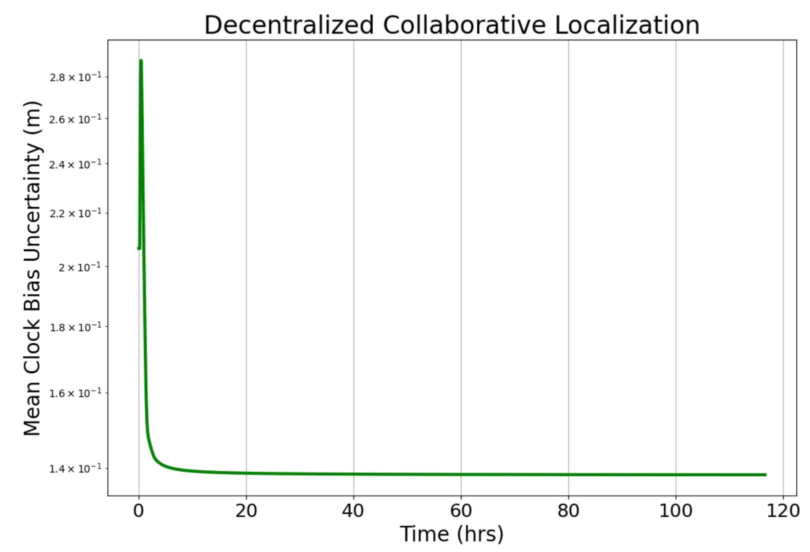


Figure 3. DCL performance using optimized communication order and tuning.

As we can see, combining both optimizations leads to rapid convergence and uncertainties comparable to the centralized Kalman filter.

Future Work

- Generalize this approach to positioning and navigation.
- Explore behavior for more complicated constellation topologies.
- Simulate and compare algorithms other than DCL.

References

[1] Lukas Luft, Tobias Schubert, Stergios I Roumeliotis, and Wolfram Burgard. Recursive decentralized collaborative localization for sparsely communicating robots. In *Robotics: Science and Systems*. New York, 2016.

For more information on our project, reach out to us at: dunn23s@mtholyoke.edu.