Hopping Forcing

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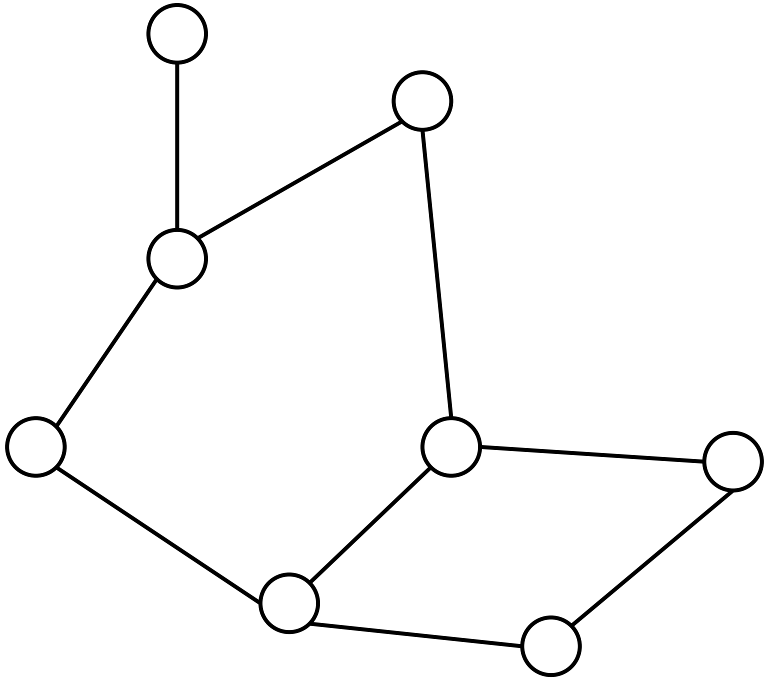
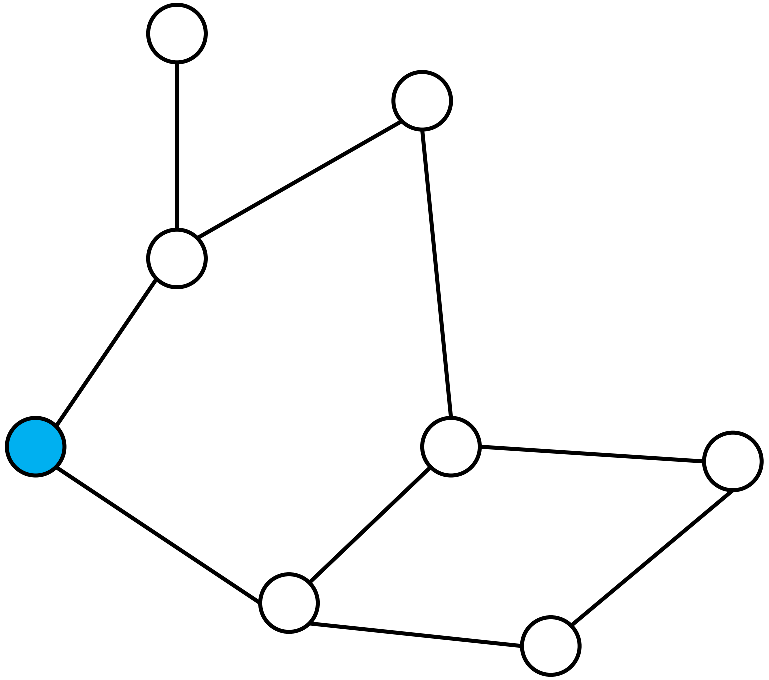
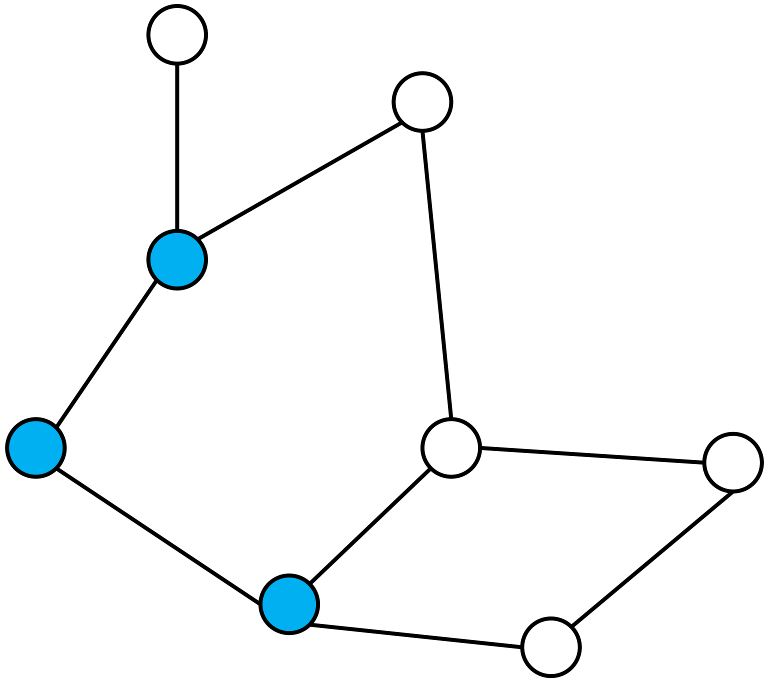
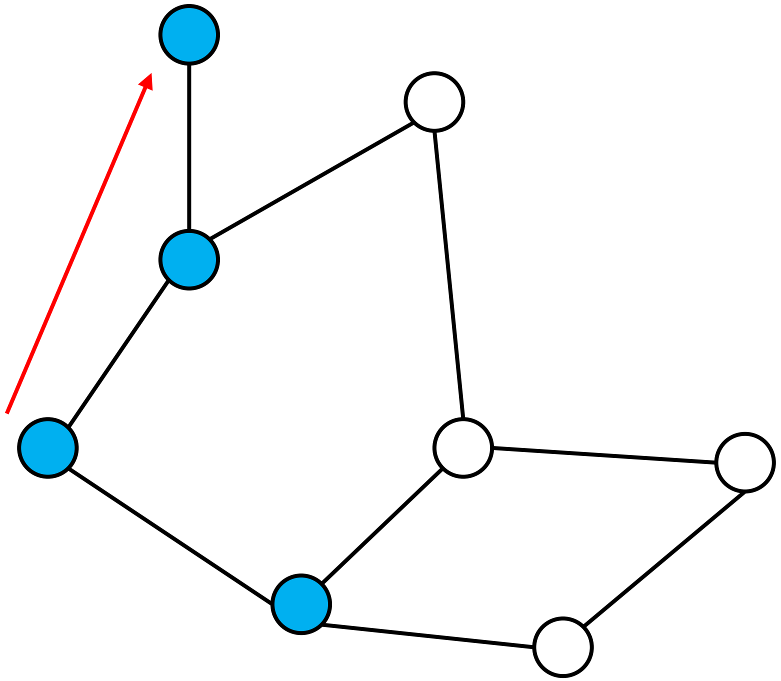
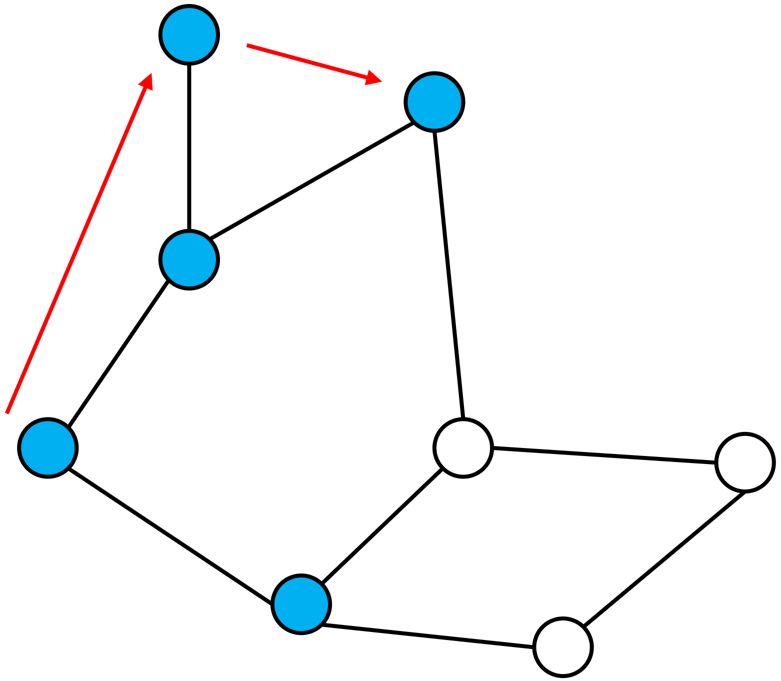
NEBRASKA CONFERENCE FOR UNDERGRADUATE WOMEN IN MATHEMATICS

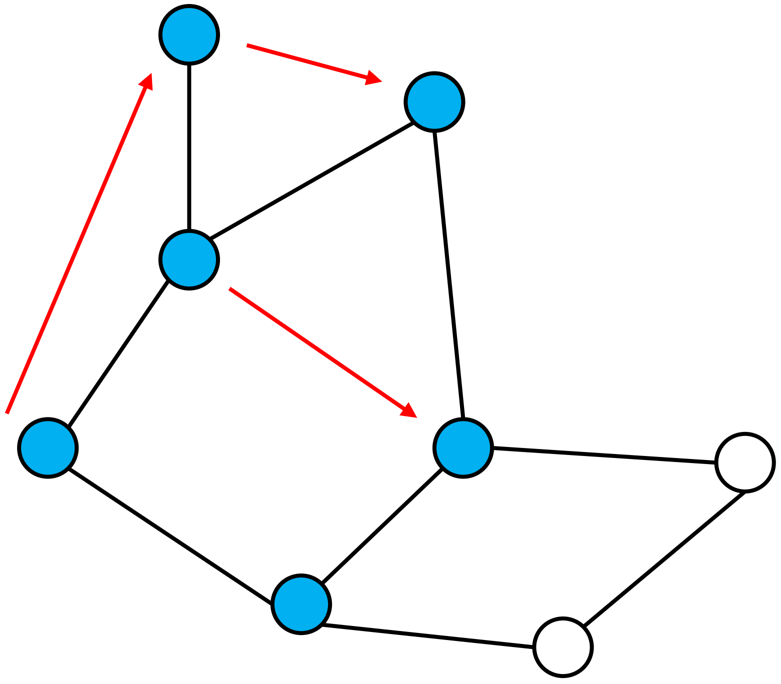


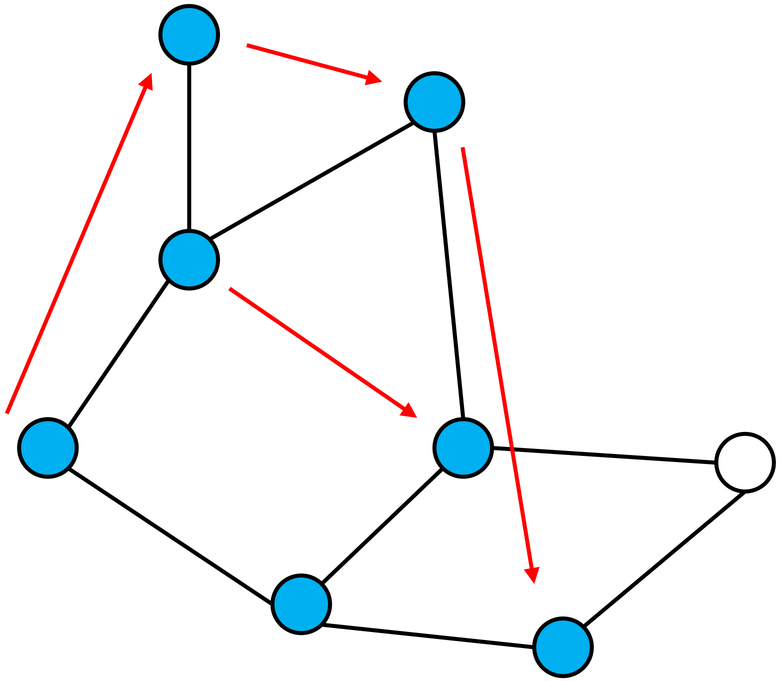


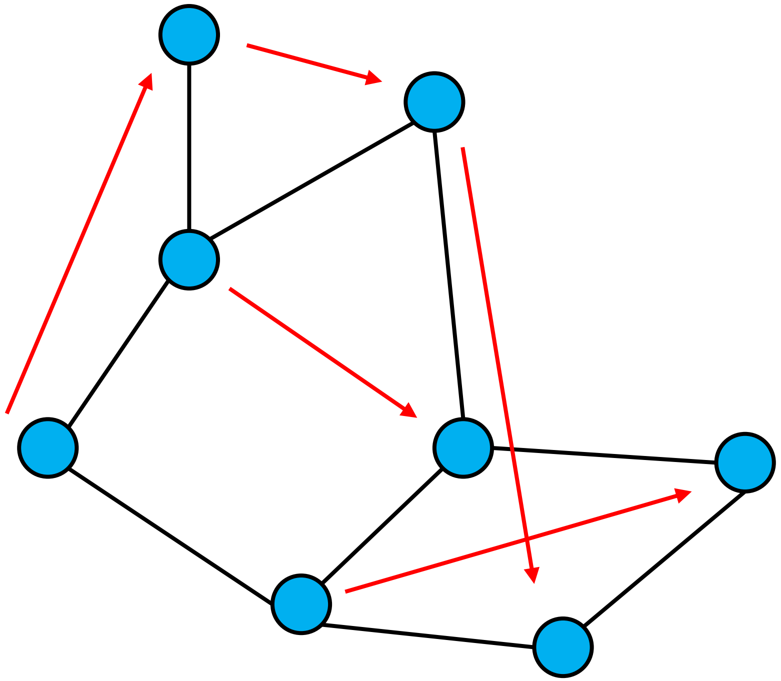












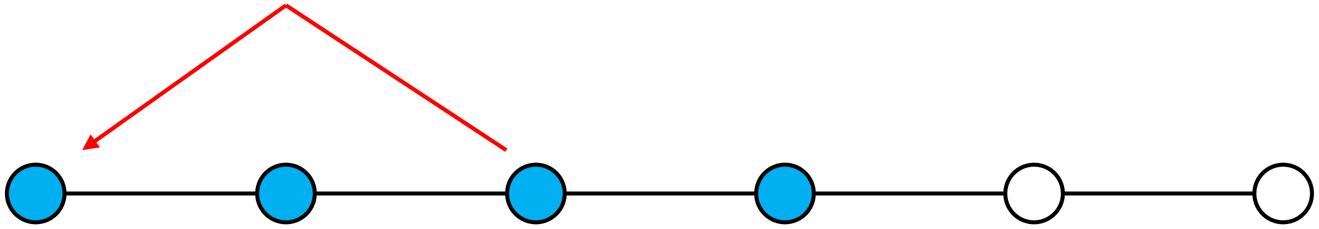
Important Terms

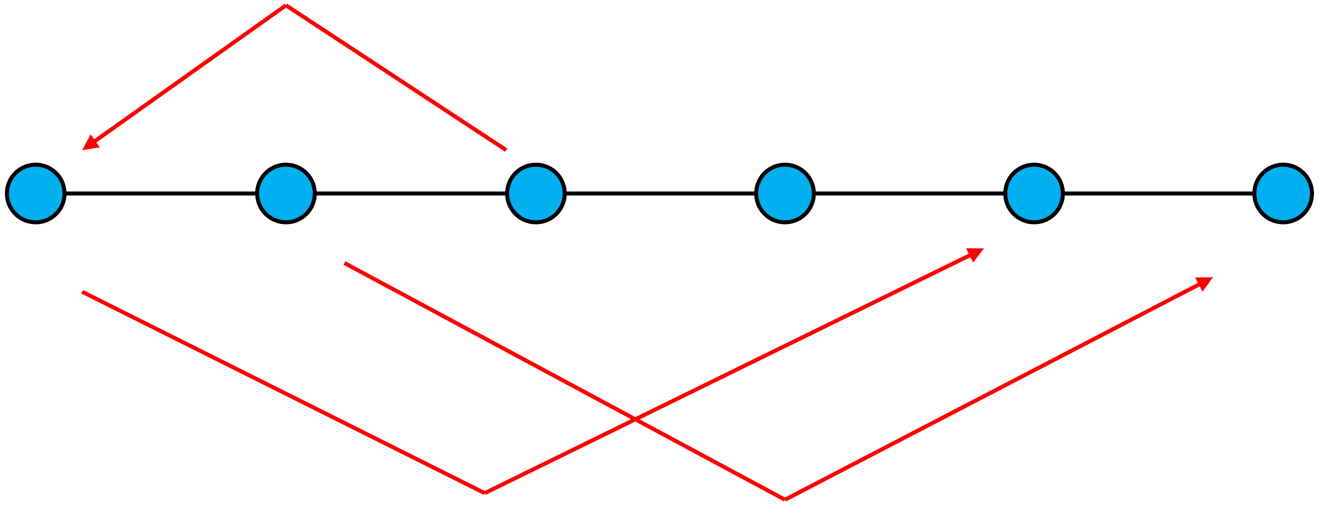
Hopping Forcing Set

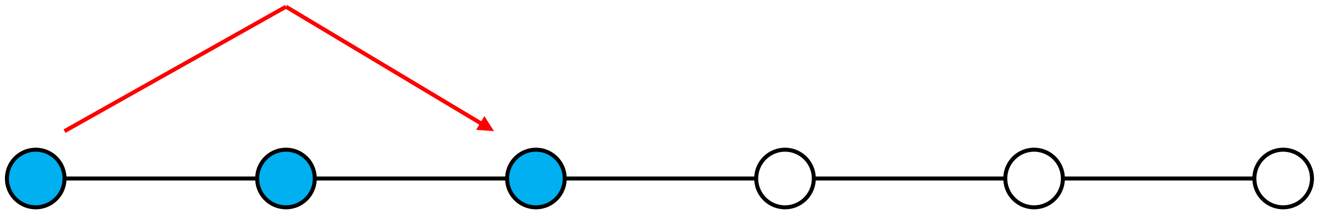
Minimum Hopping Forcing Set (MHFS)

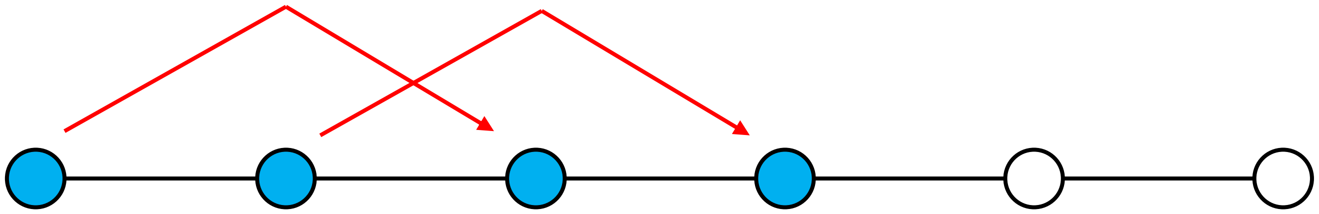
Hopping Forcing Number

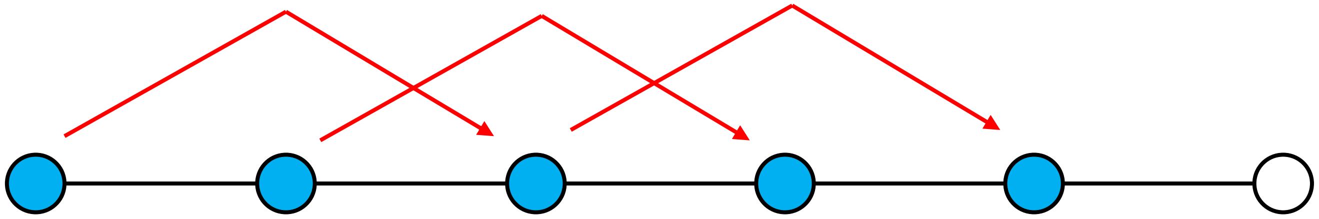
Propagation Time

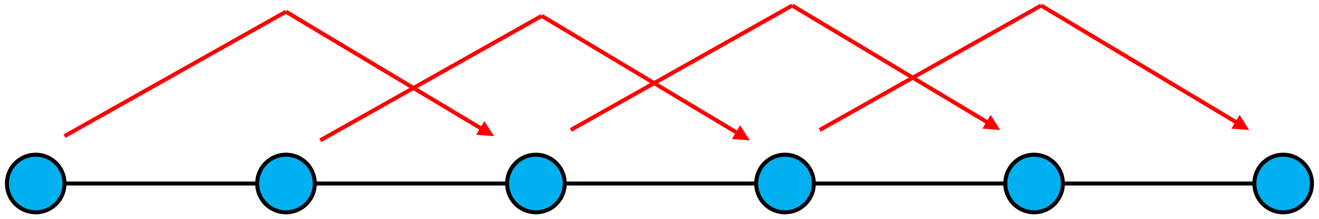








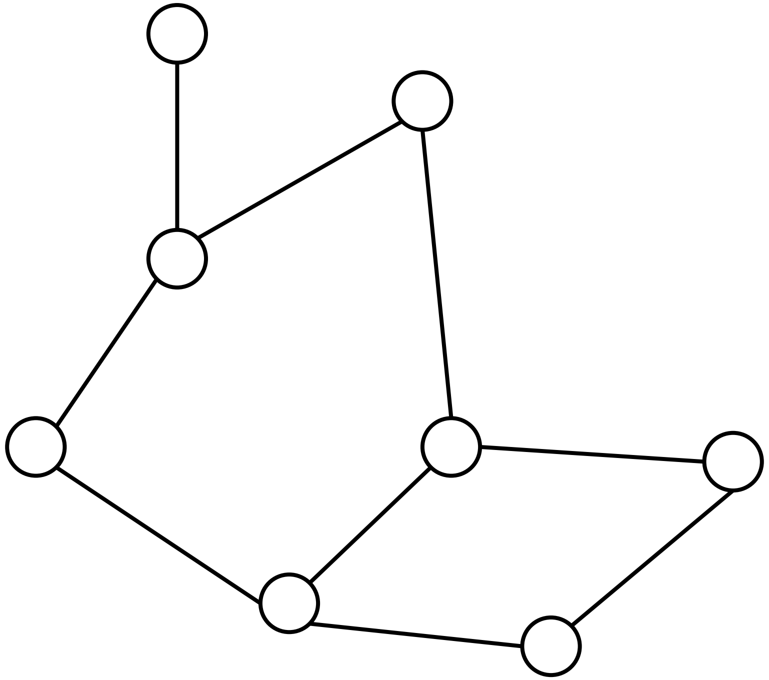




Results

Proposition 2.1. *The hopping forcing number is subgraph monotone (i.e., if G_1 is a subgraph of G_2, then $H(G_1) \leq H(G_2)$).*

Hopping number of G1



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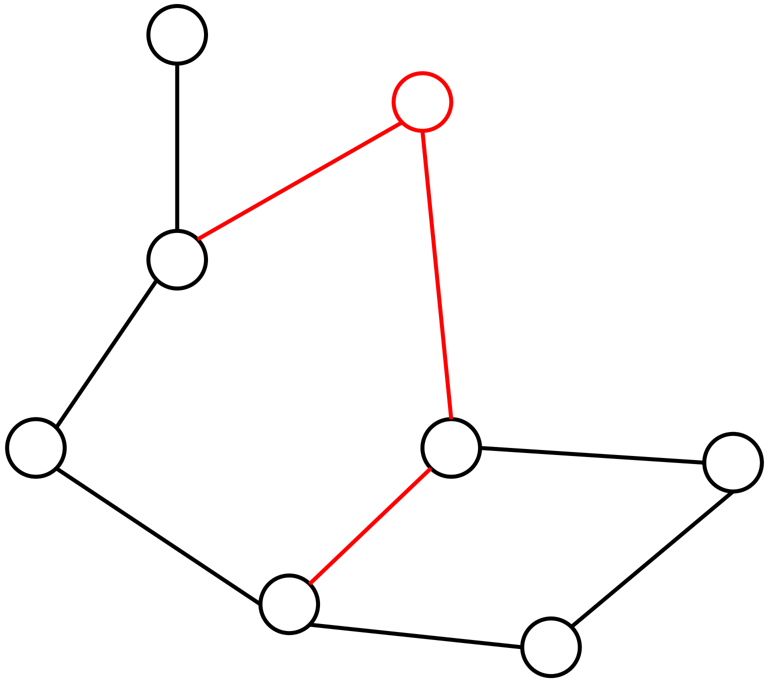

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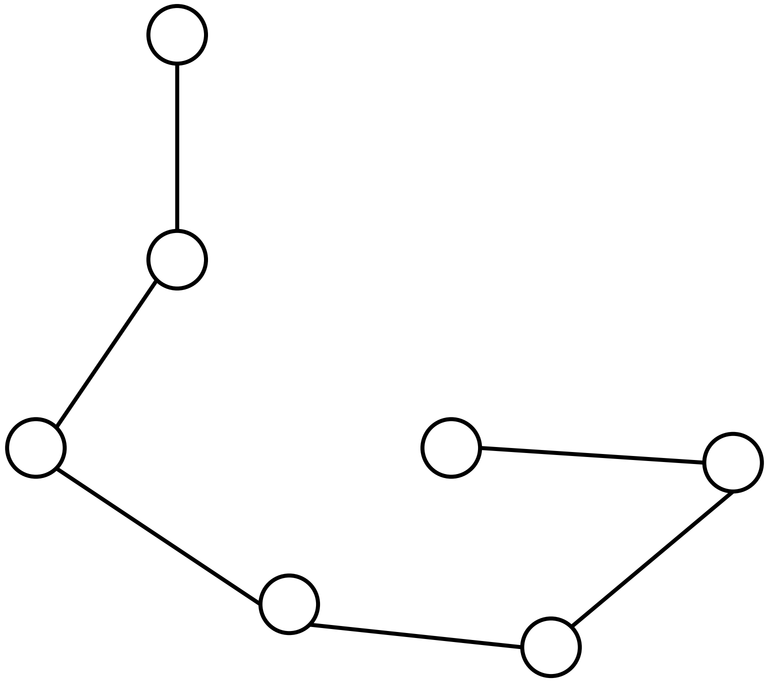

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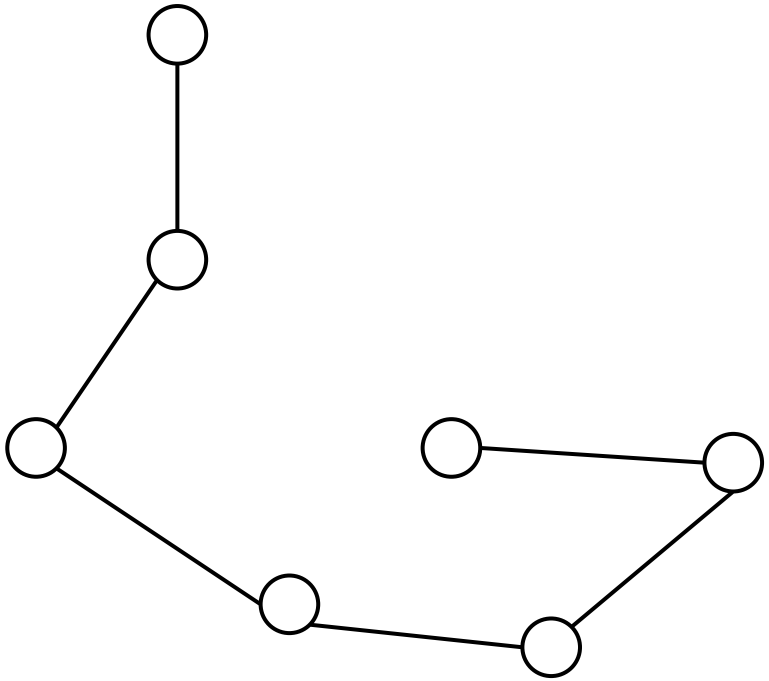
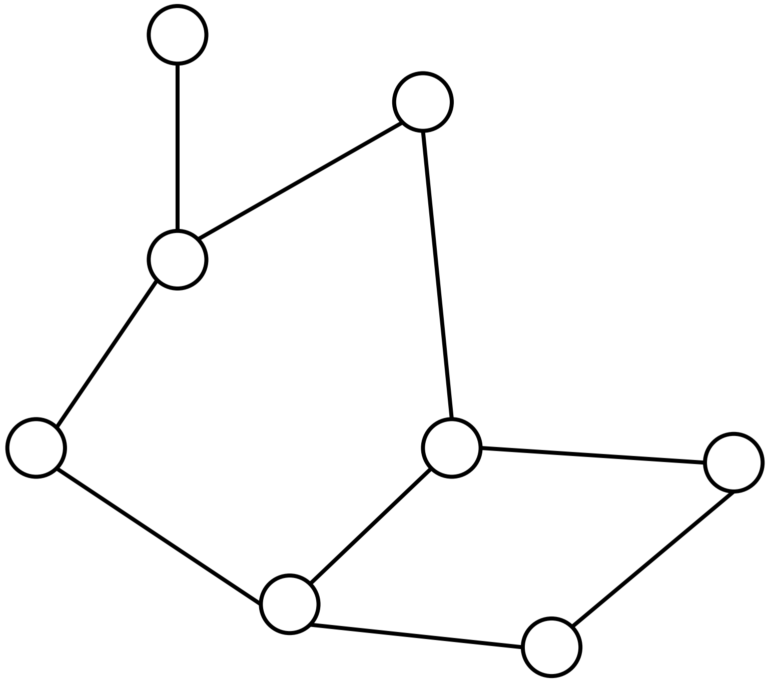
G1



G2



Proof Method

Compare the set of forces for the original graph to the subgraph .

- Every set of forces that works in G2, will work in G1.
  - Deleting edges cannot make an active vertex become unactive.

Suppose there exists another subgraph of G2, call it G3 and it is missing a vertex.

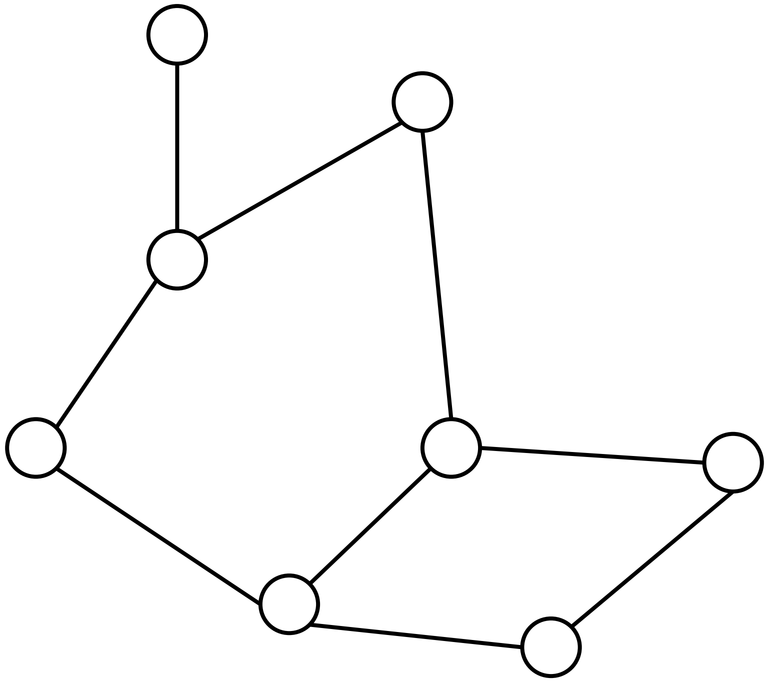
Two cases: the missing vertex was in the minimum hopping forcing set of G3, or it was not.

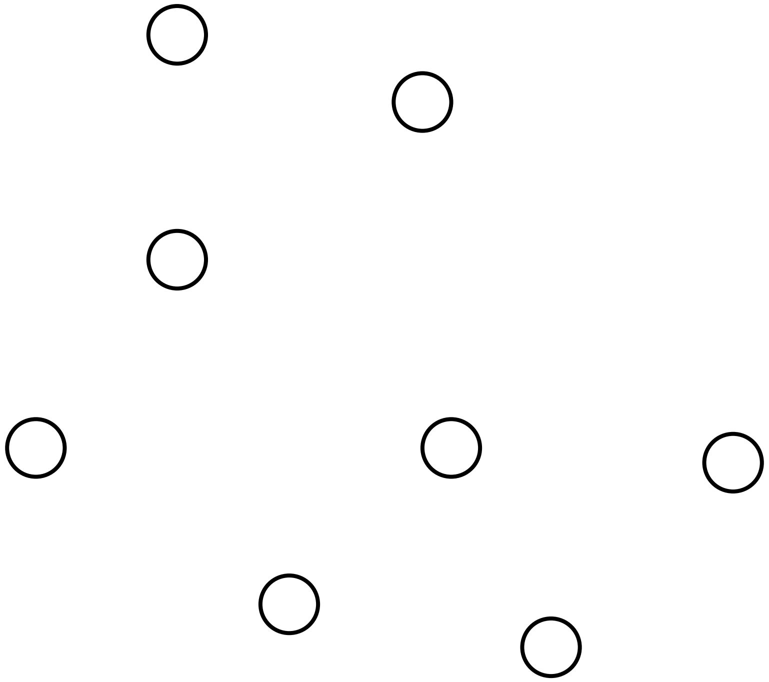
- Suppose missing vertex in MHFS:
  - Perform a force: Then MHFS and the forced vertex, minus the missing vertex is a MHFS.
  - Not perform a force: Then the MHFS minus the missing vertex is a hopping forcing set.
- Suppose missing vertex not in MHFS:
  - Perform a force: Then the MHFS is the same.
  - Not perform a force: The MHFS is the same, and the set of forces changes. (i.e., taking out the middleman)

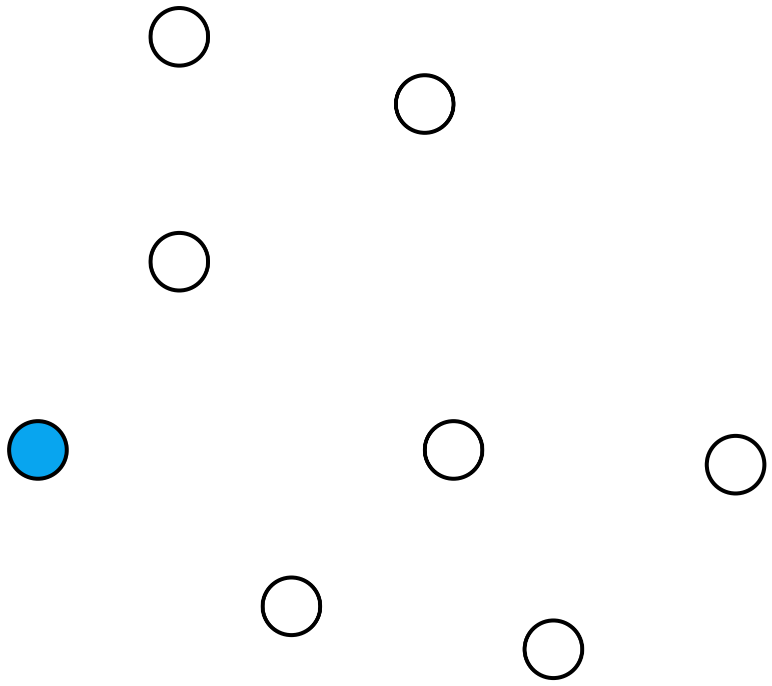
For every case, the size of the MHFS of the subgraph is less than or equal to the size of the MHFS of the original graph.

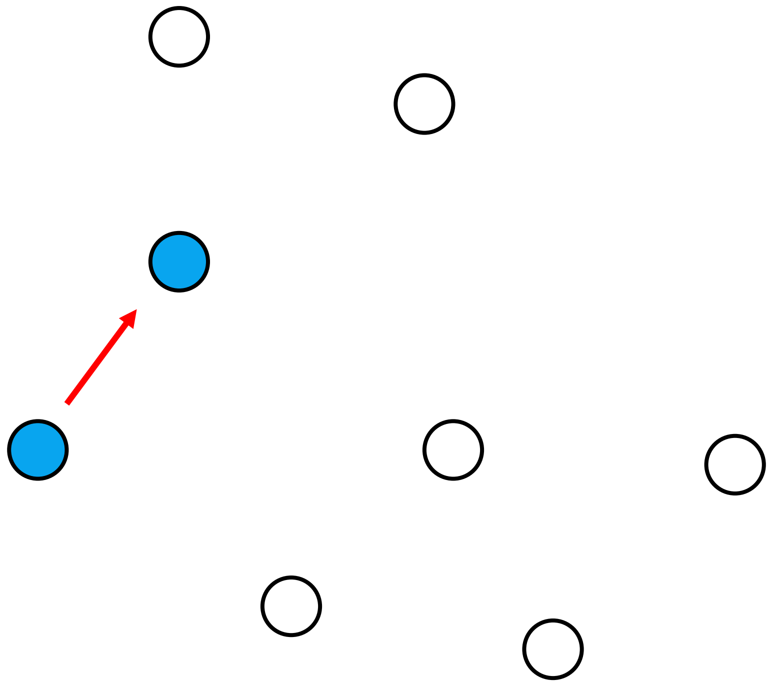
Thus, the hopping number of the subgraph is less than or equal to hopping number of the original graph.

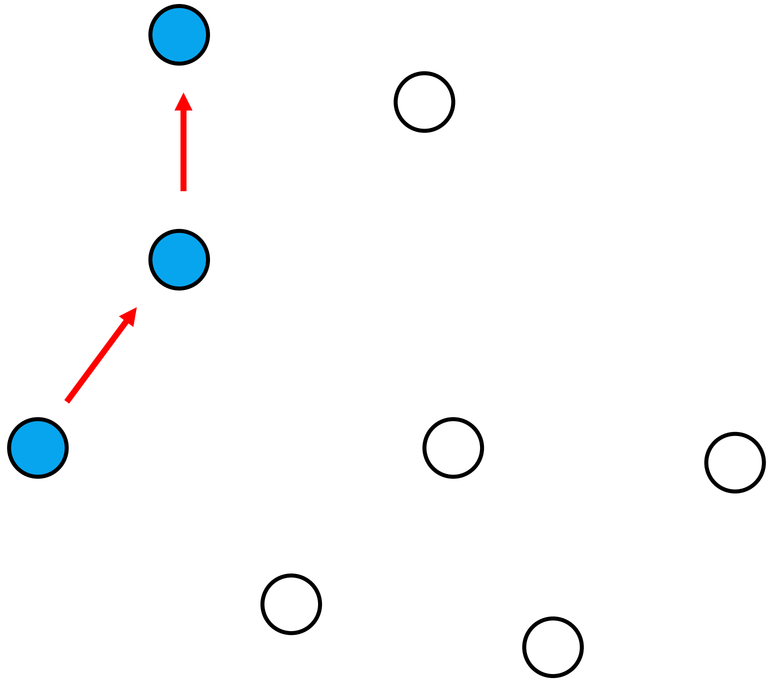
Recall: Hopping number is 3

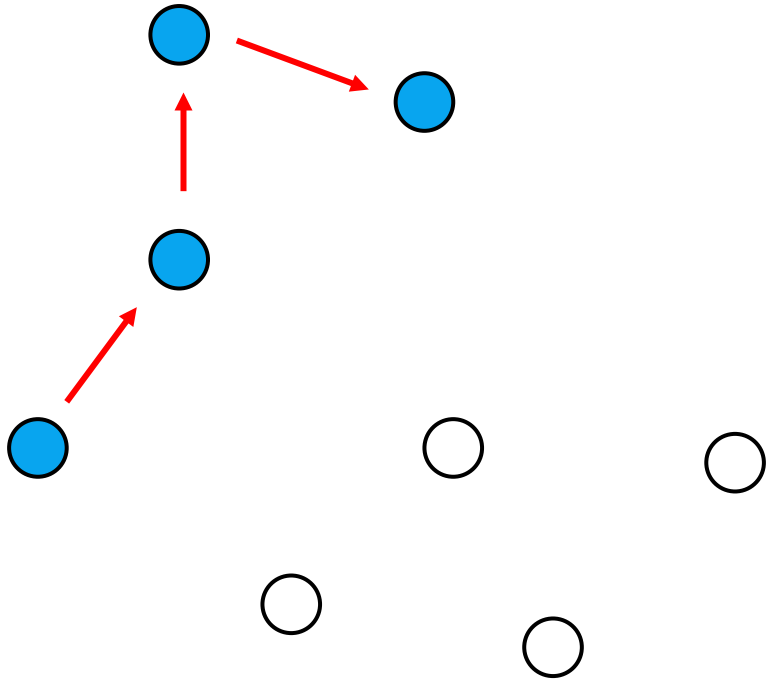


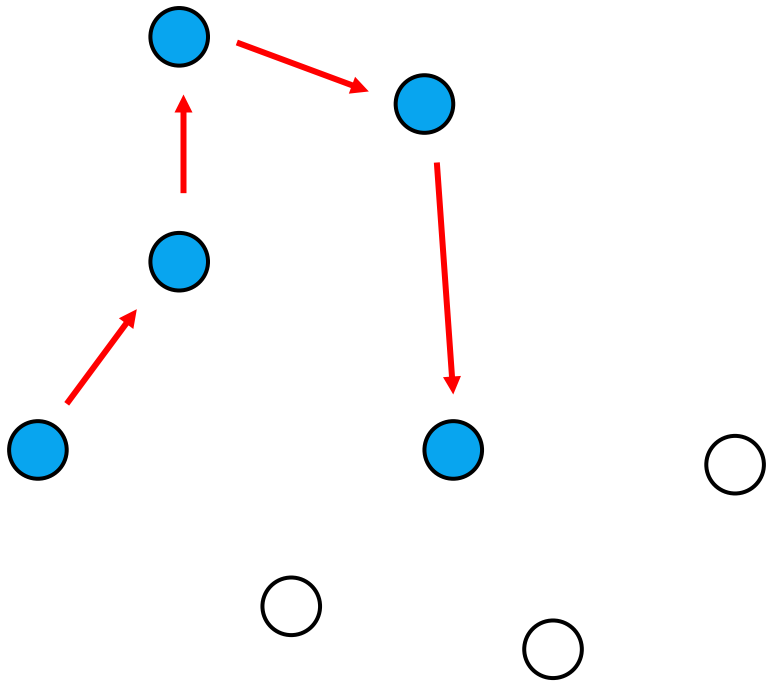


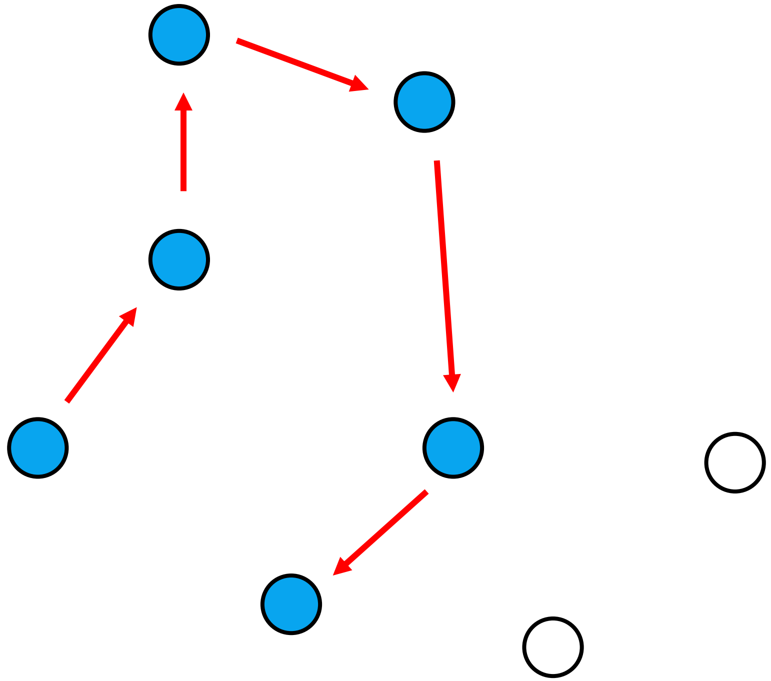


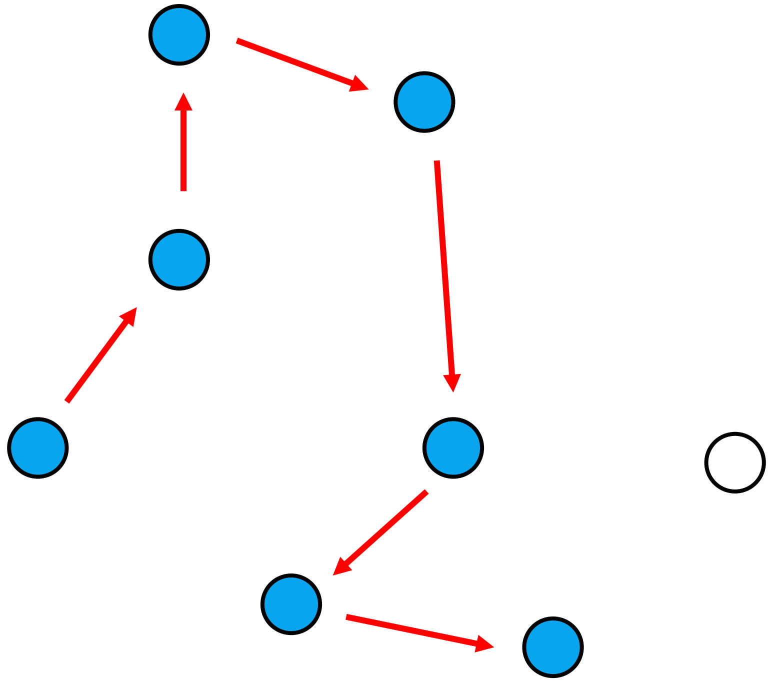




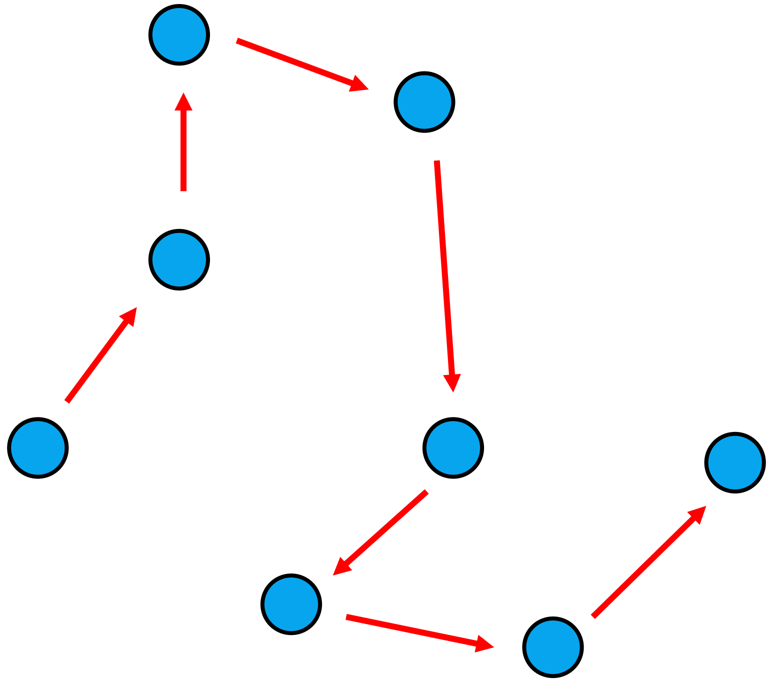








Note: Hopping number is 1



Proof Method

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=<

Suppose there exists another subgraph of G2, call it G3 and it is missing a vertex.

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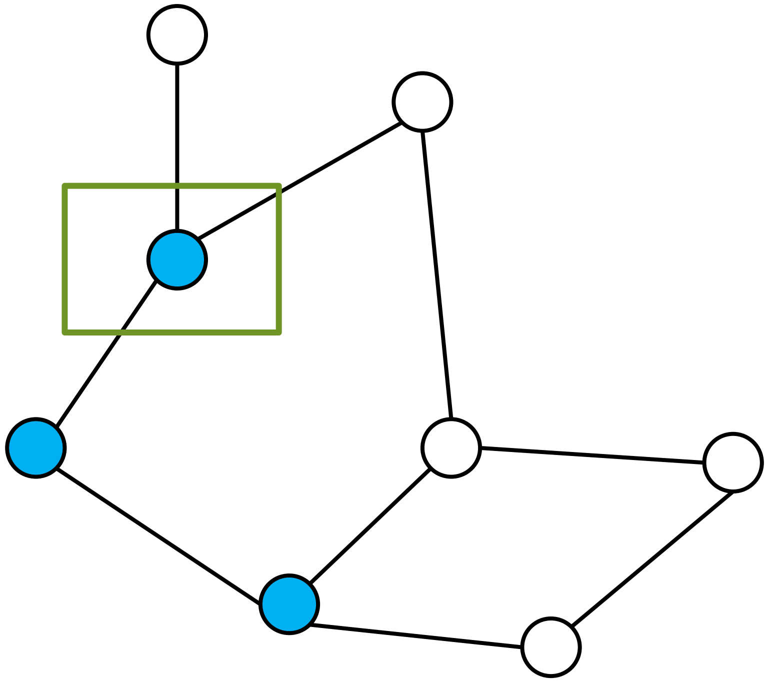
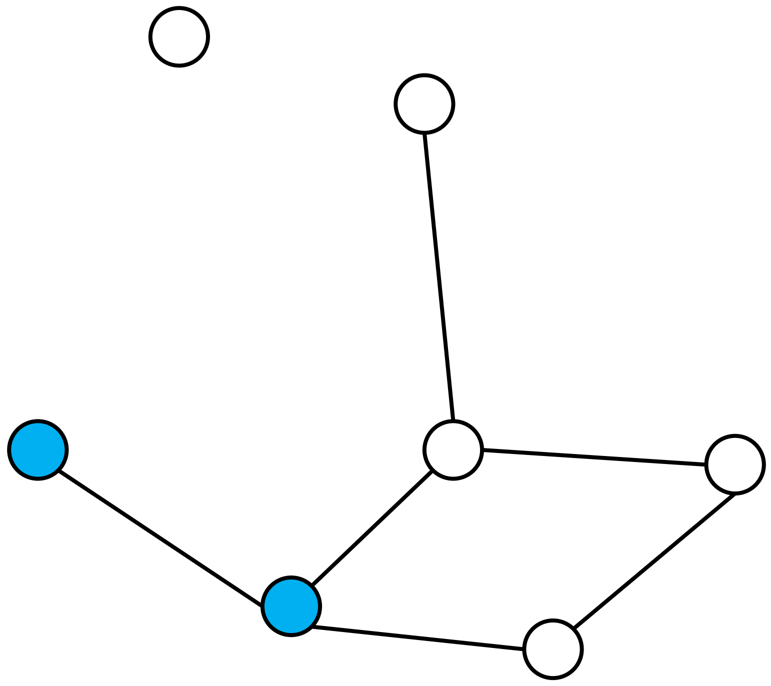
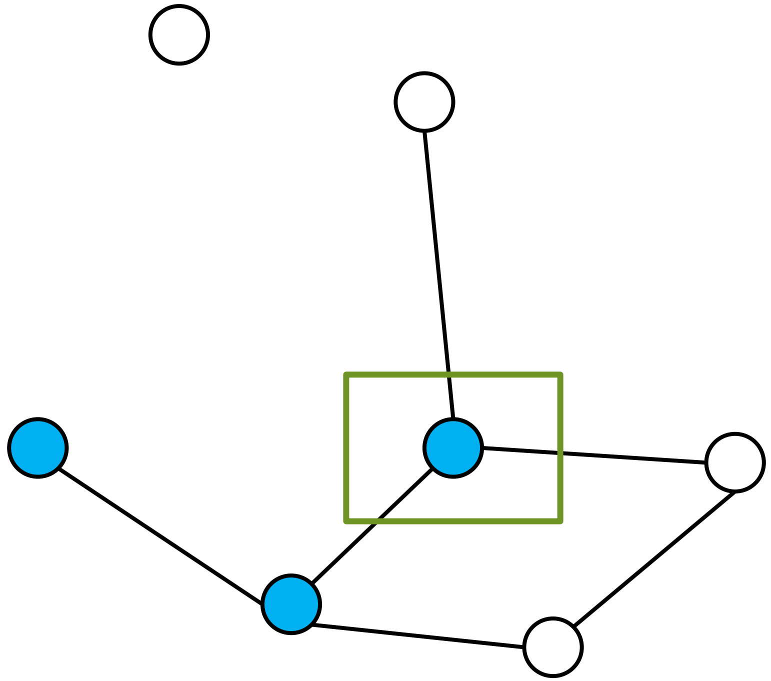
Thus, the hopping number of the subgraph is less than or equal to hopping number of the original graph.





Note: Removed a vertex from MHFS AND added a vertex to MHFS



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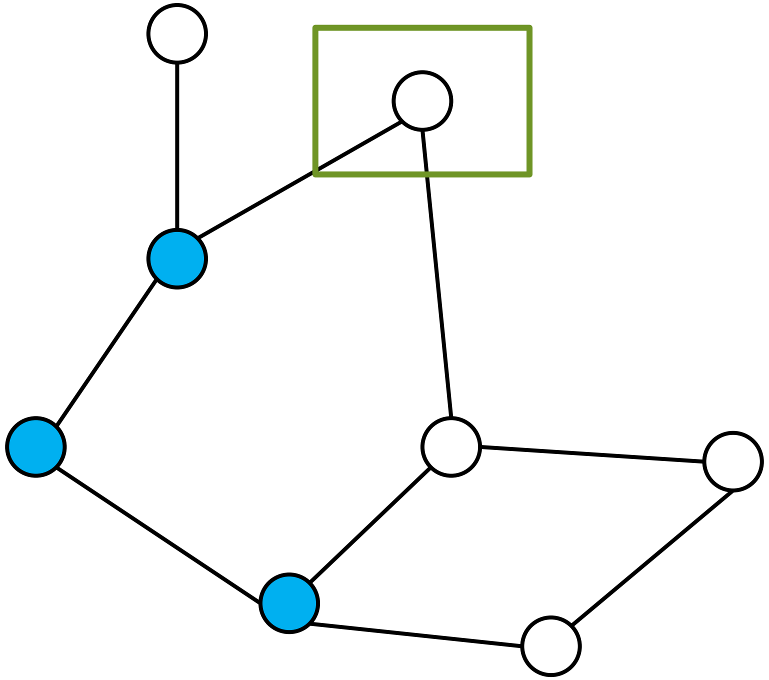
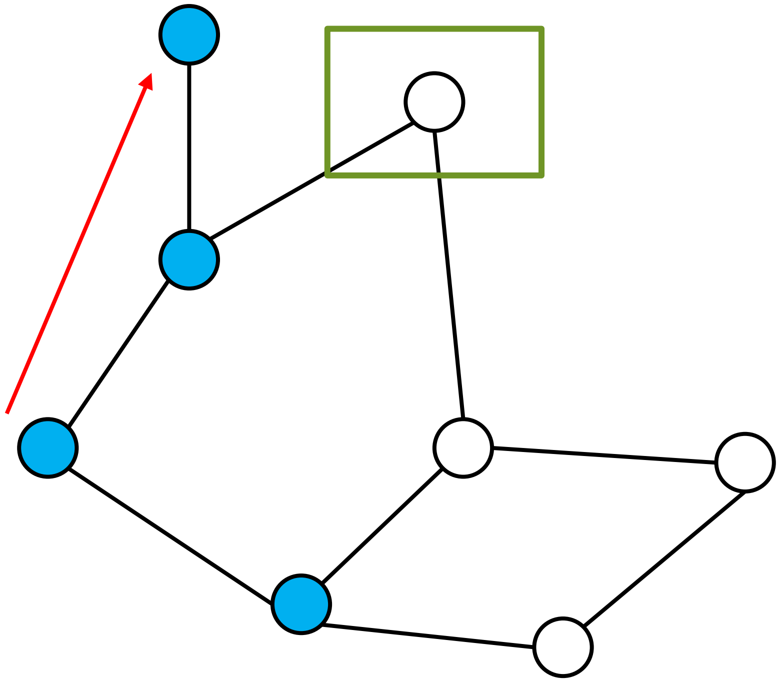
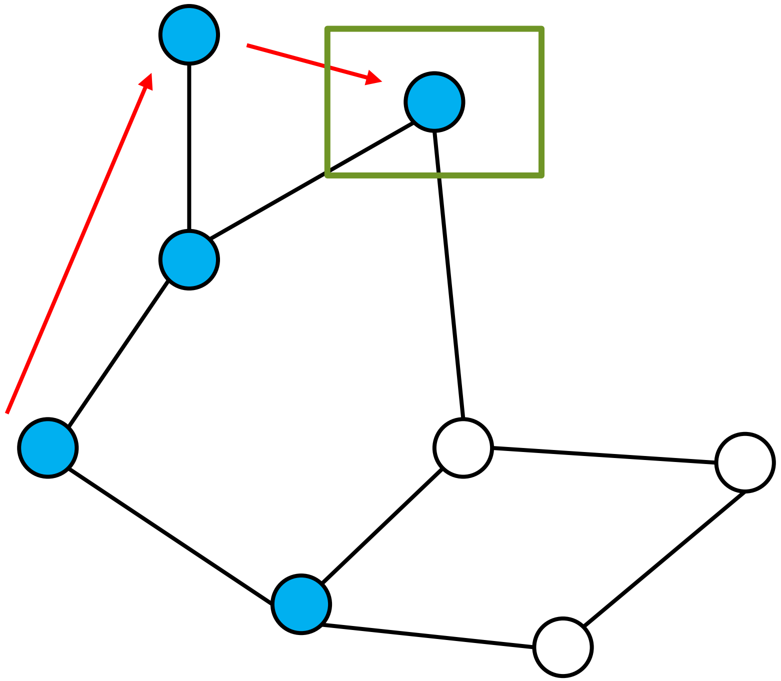
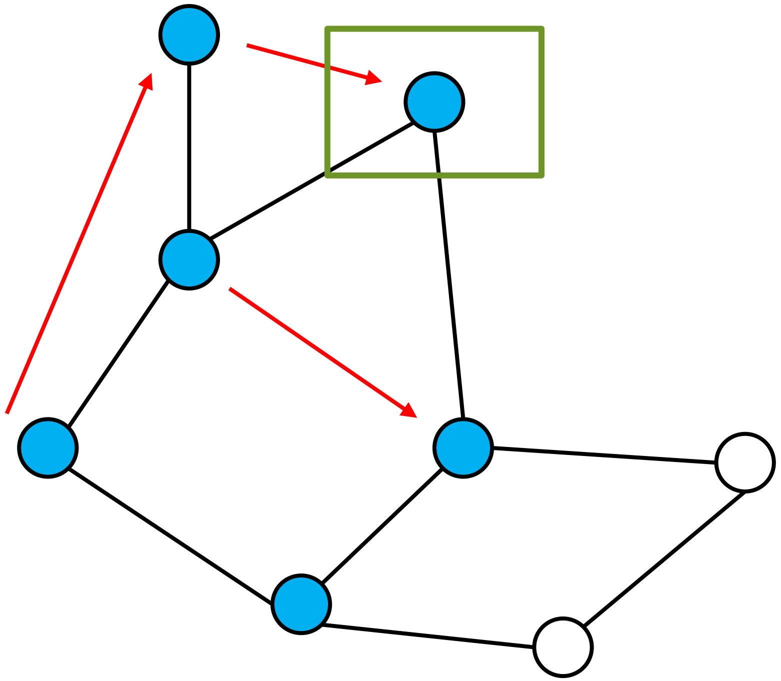
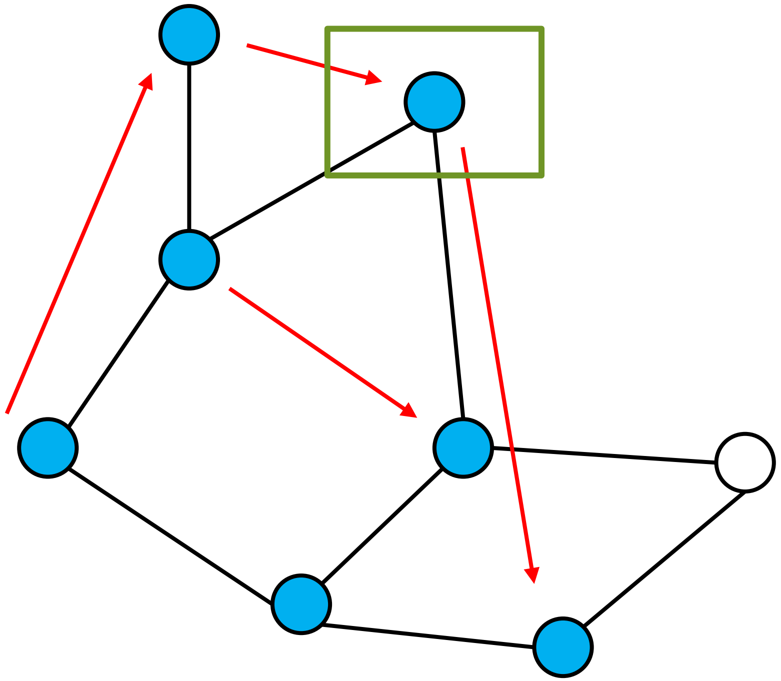
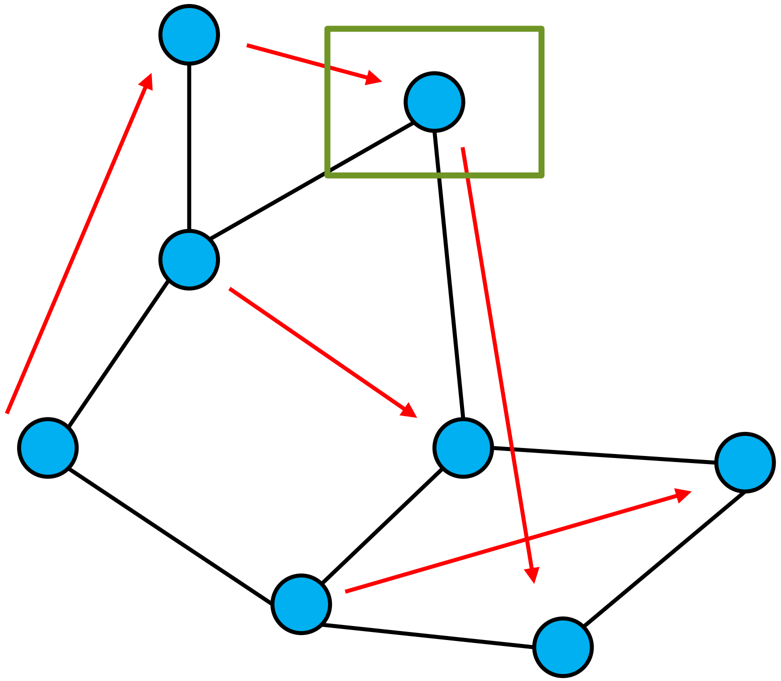


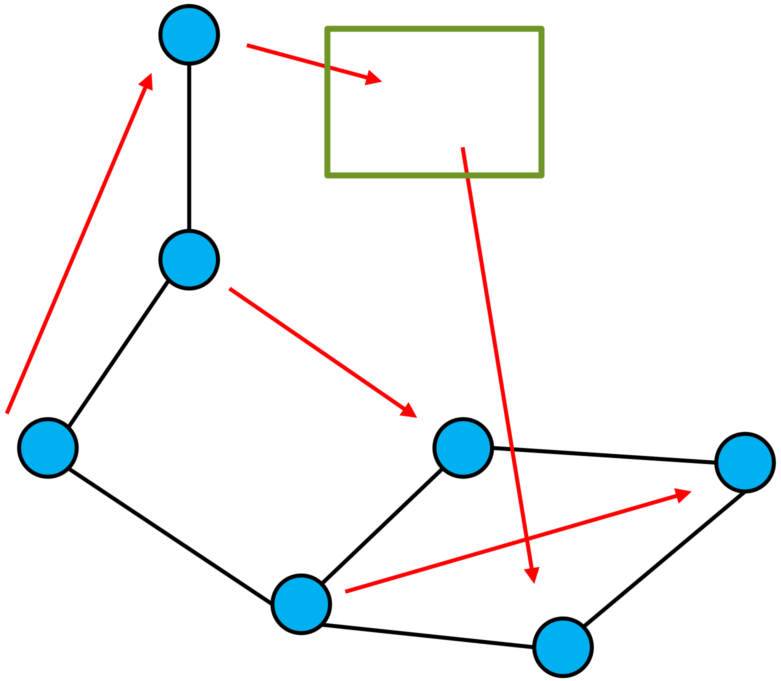




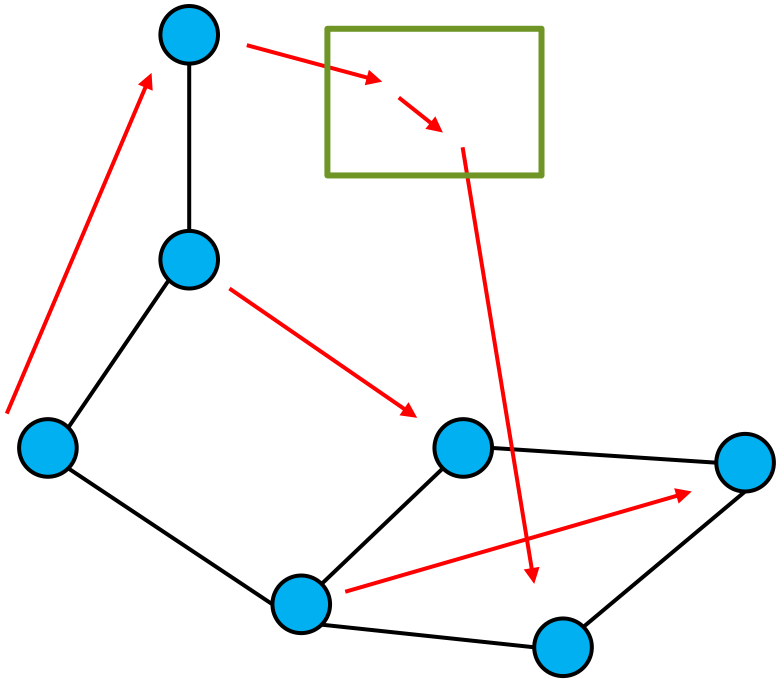








Note: Reroute!!



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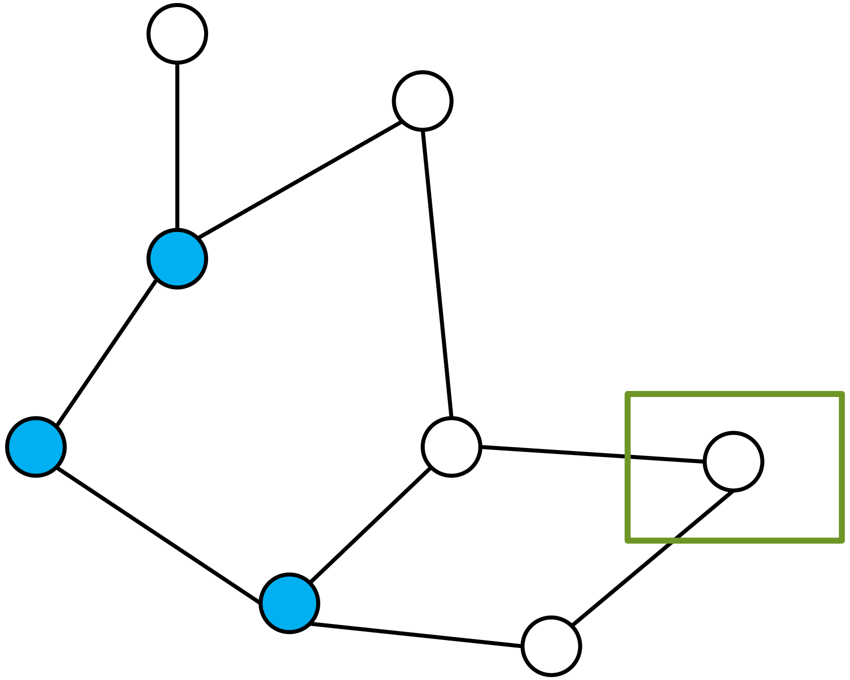
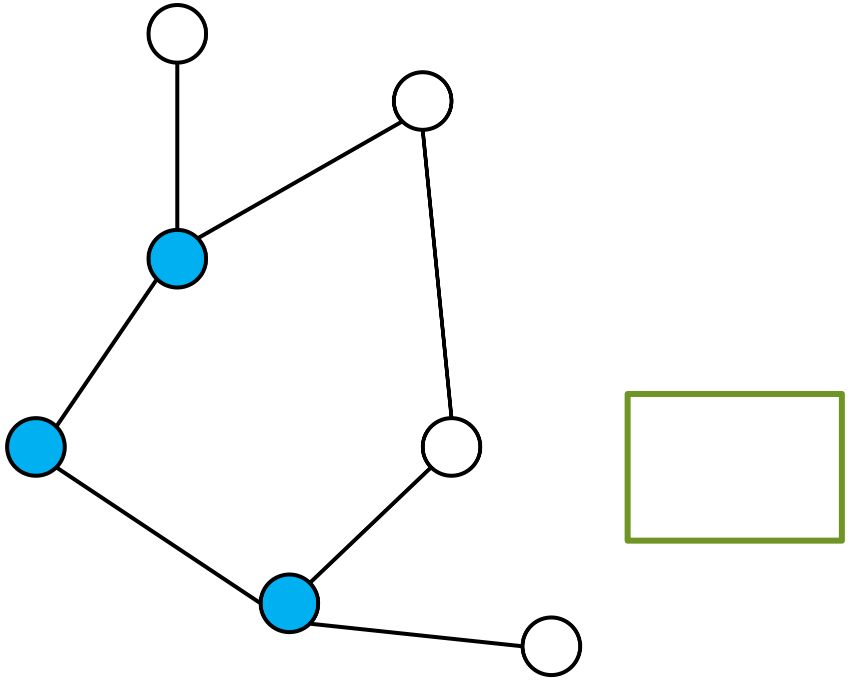
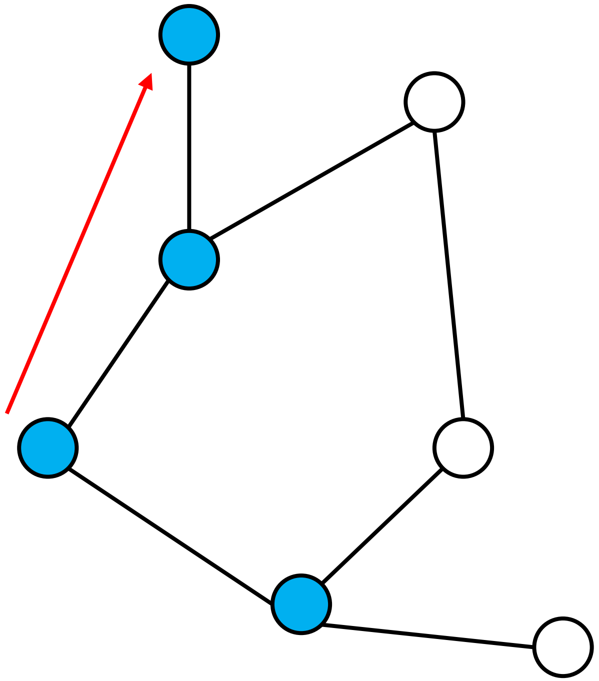
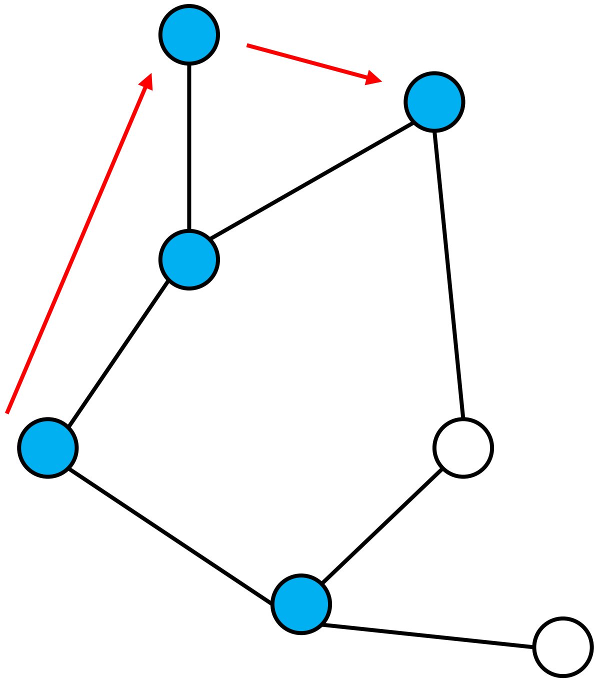
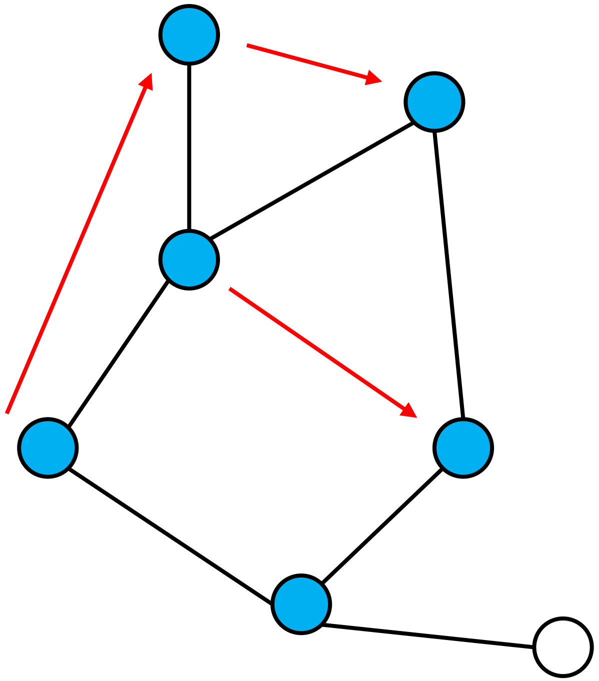


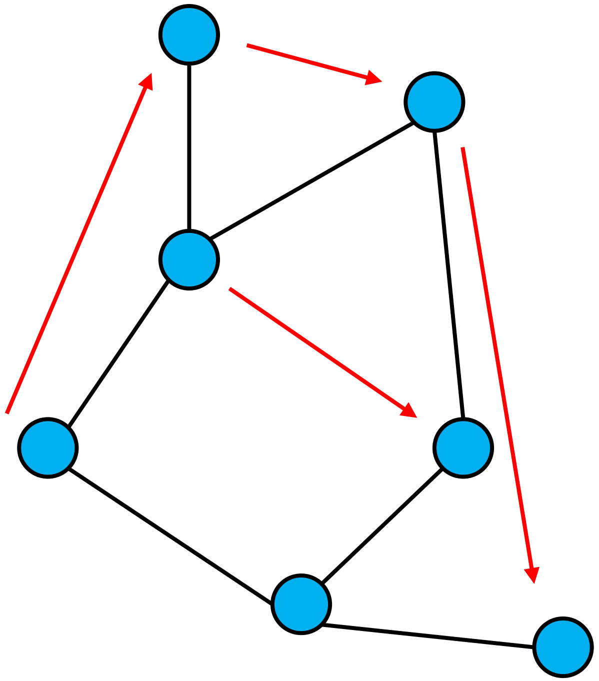








Note: MHFS stayed the same



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Questions?