The Safety Net Problem: A Variation on Graph Theory's Minimum Spanning Tree (MST) Algorithm

for NCUWM 2023

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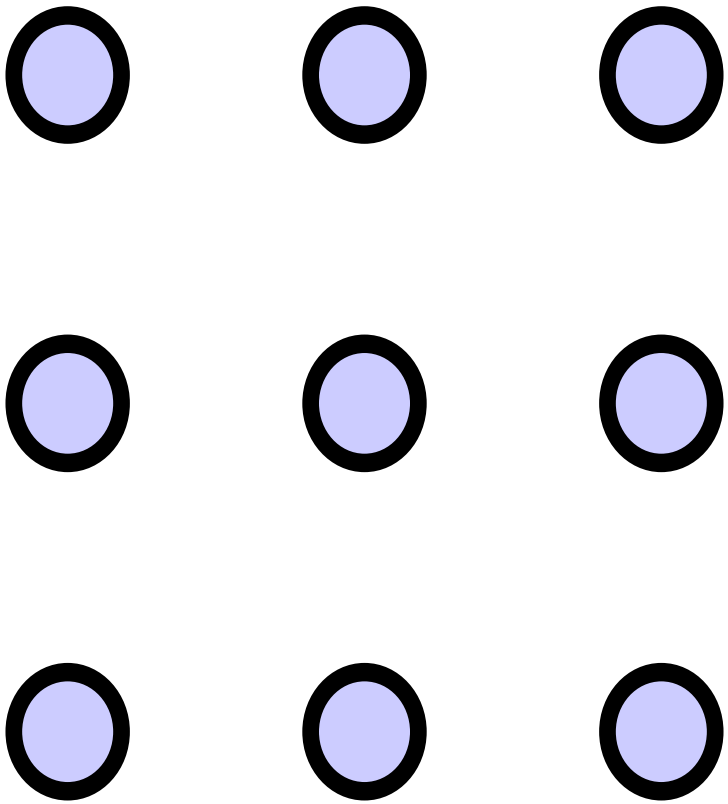
Outline

Background

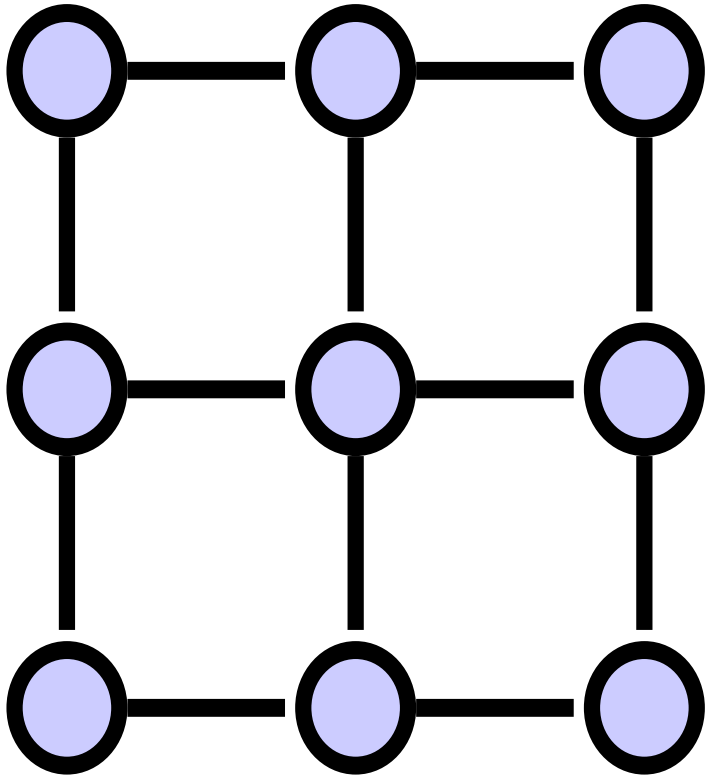
Definition

A **graph**, G, contains a set of, E, edges and V, vertices, i.e. $G = \{E, V\}$.

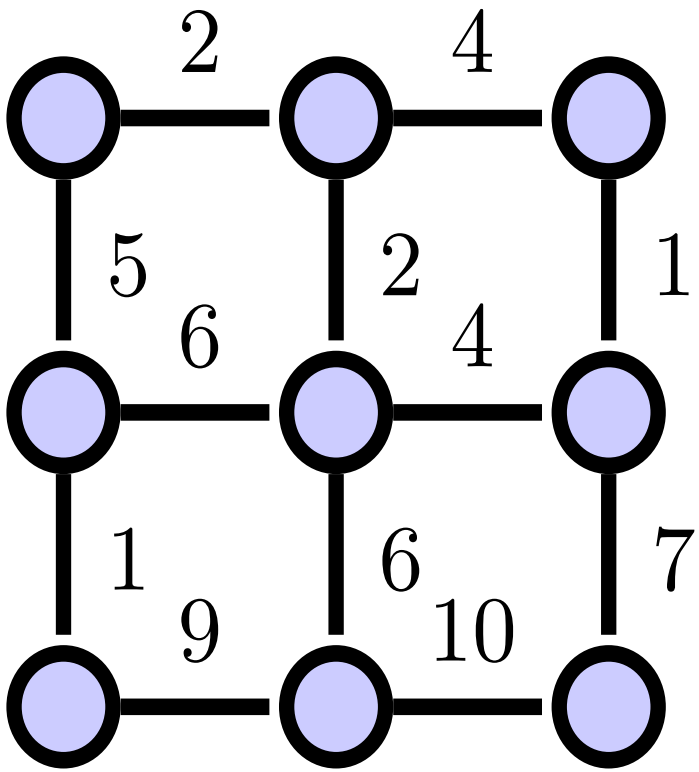
Background: Nodes



Background: Edges



Background: Weights



Outline

Problem: Minimum Spanning Tree (MST)

"Given a set of (point) terminals, connect them by a network of direct terminal-to-terminal links having the smallest possible total length (sum of the link lengths)."

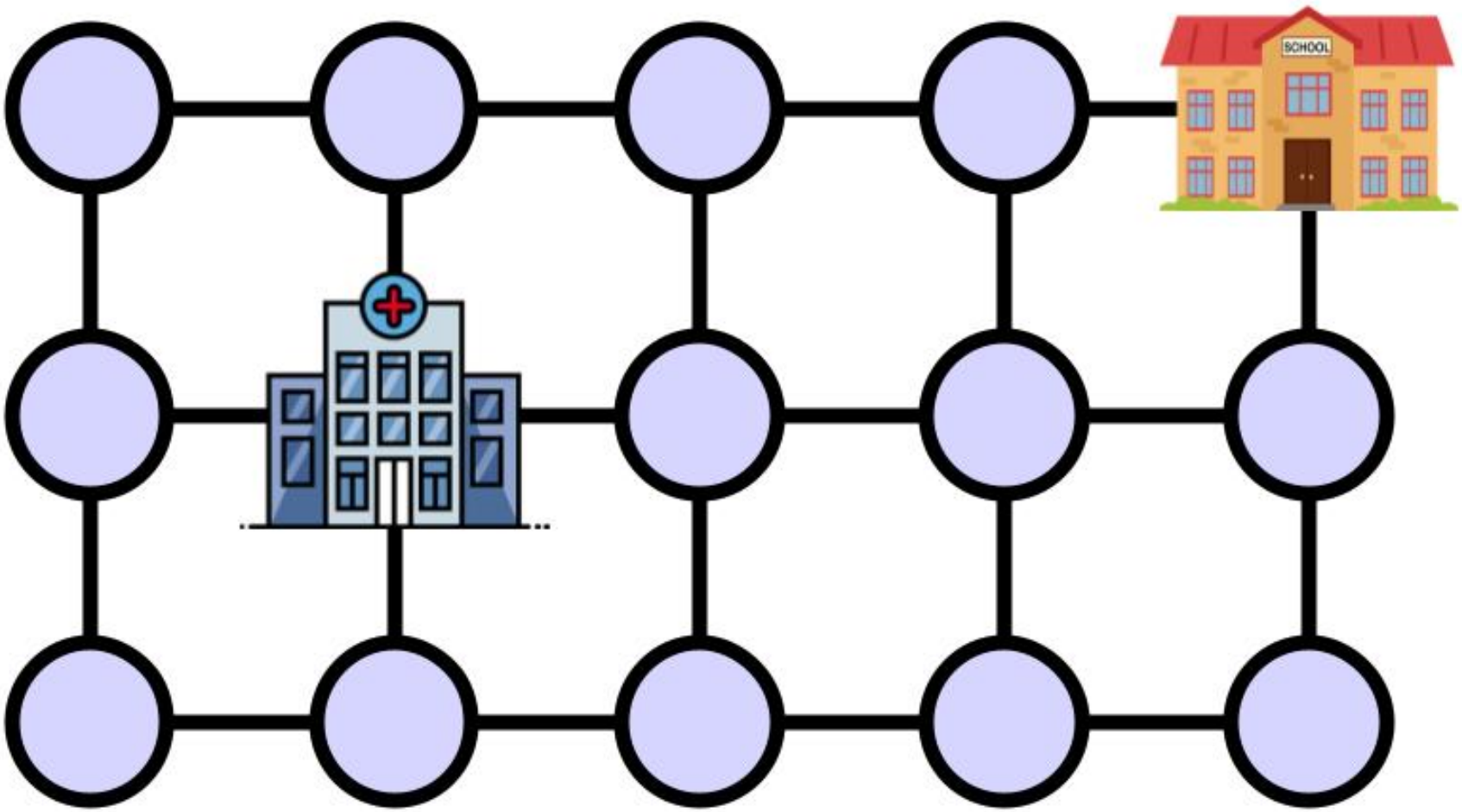
- R. C. Prim

Problem: Minimum Spanning Tree (MST)

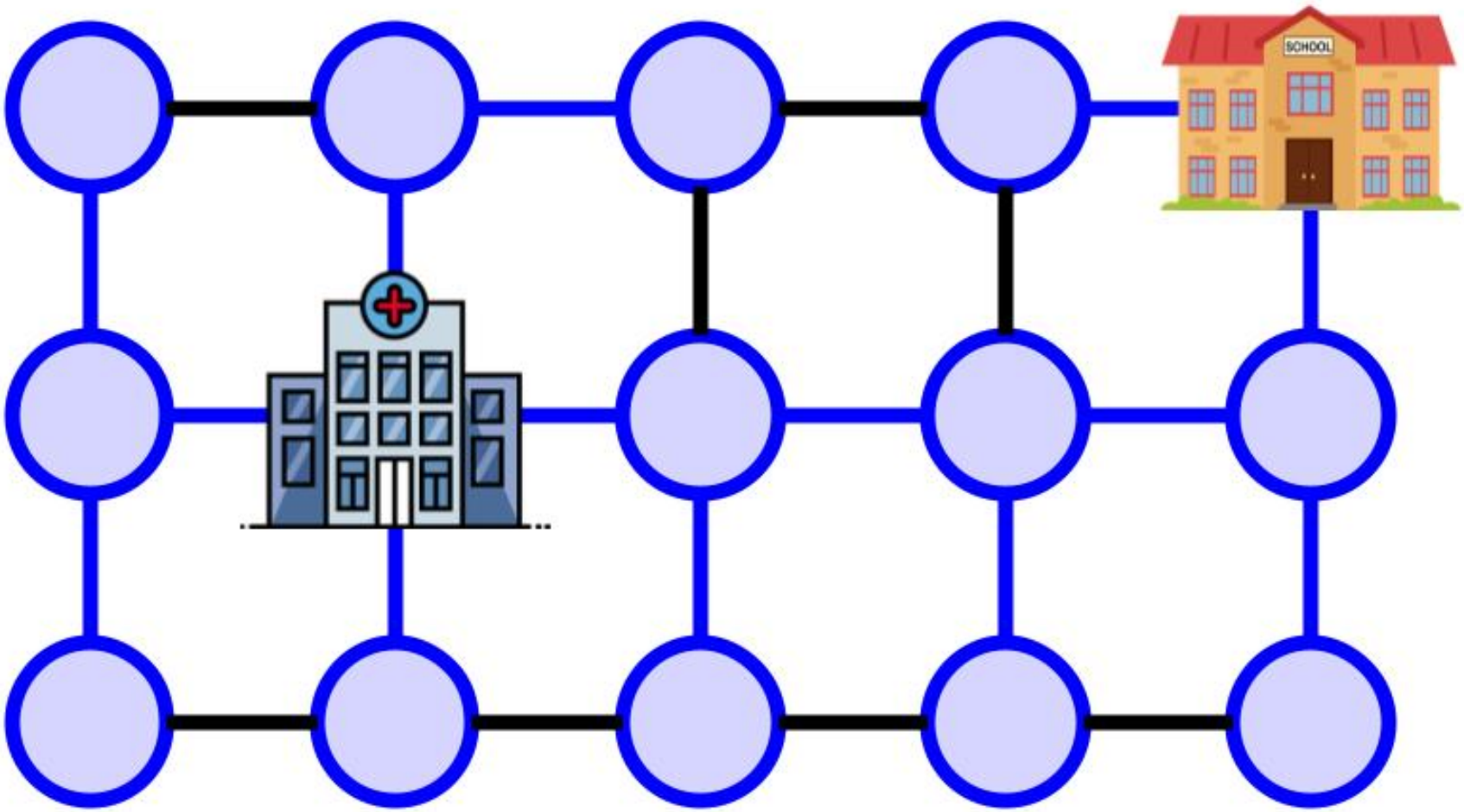
"Given a connected graph, G, with positive edge weights, find a minimum weight set of edges that connects all of the vertices."

- *GeeksforGeeks*

Application Example



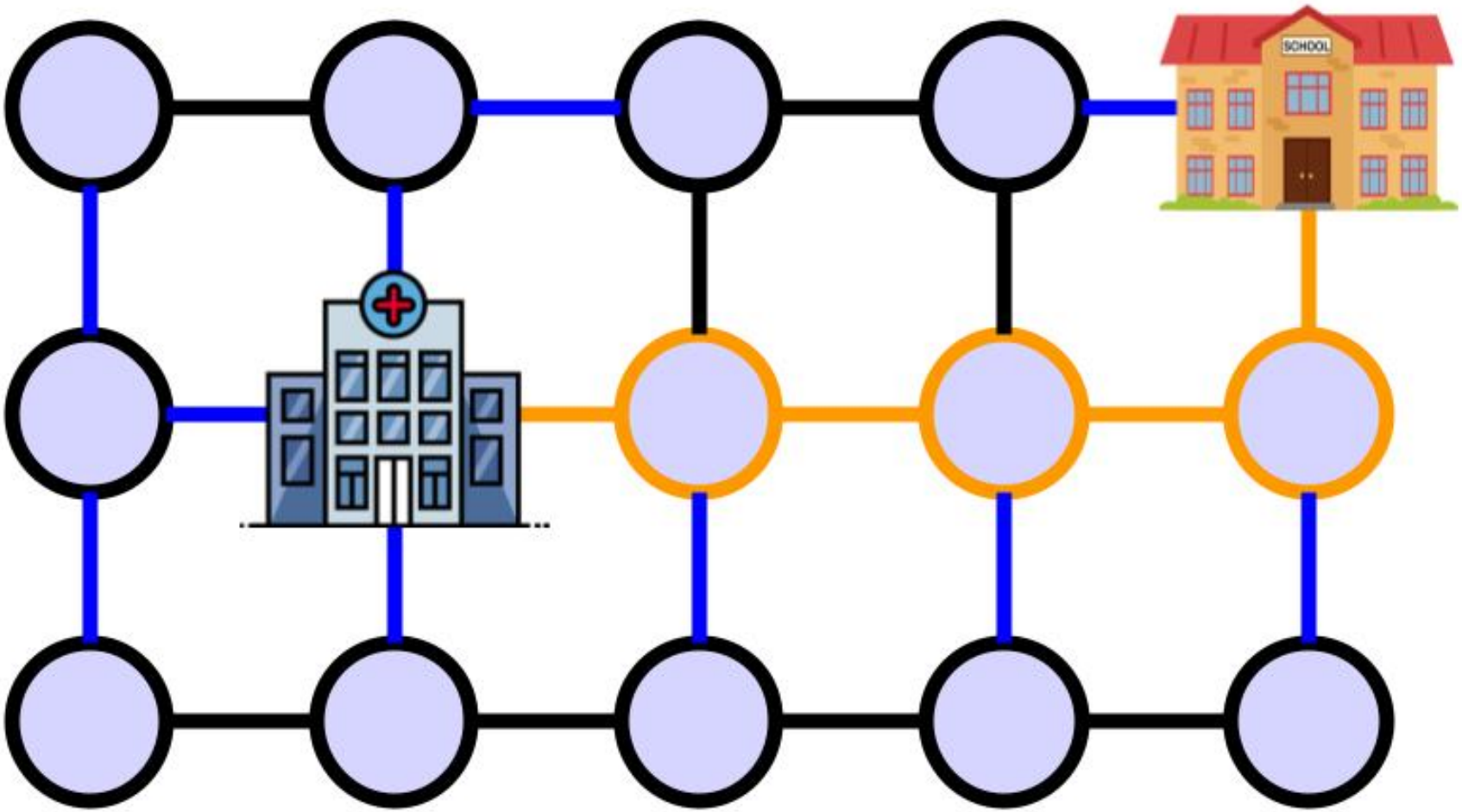
Application Example: MST



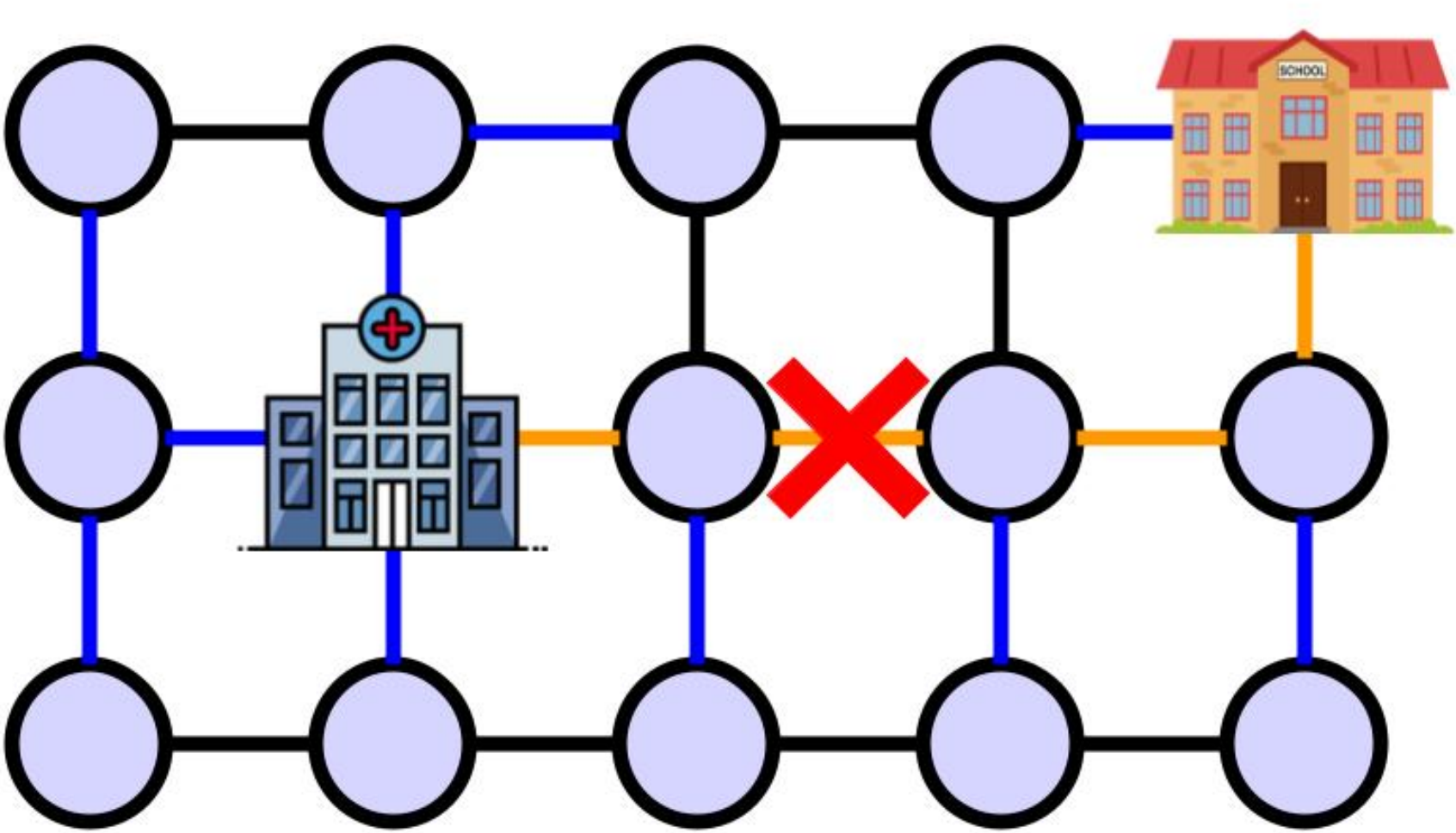
Problem: Our Addition

Additionally: How do we find the optimal path on graph, G, between required nodes, in the set R, such that if an edge is removed, the required nodes will remain connected?

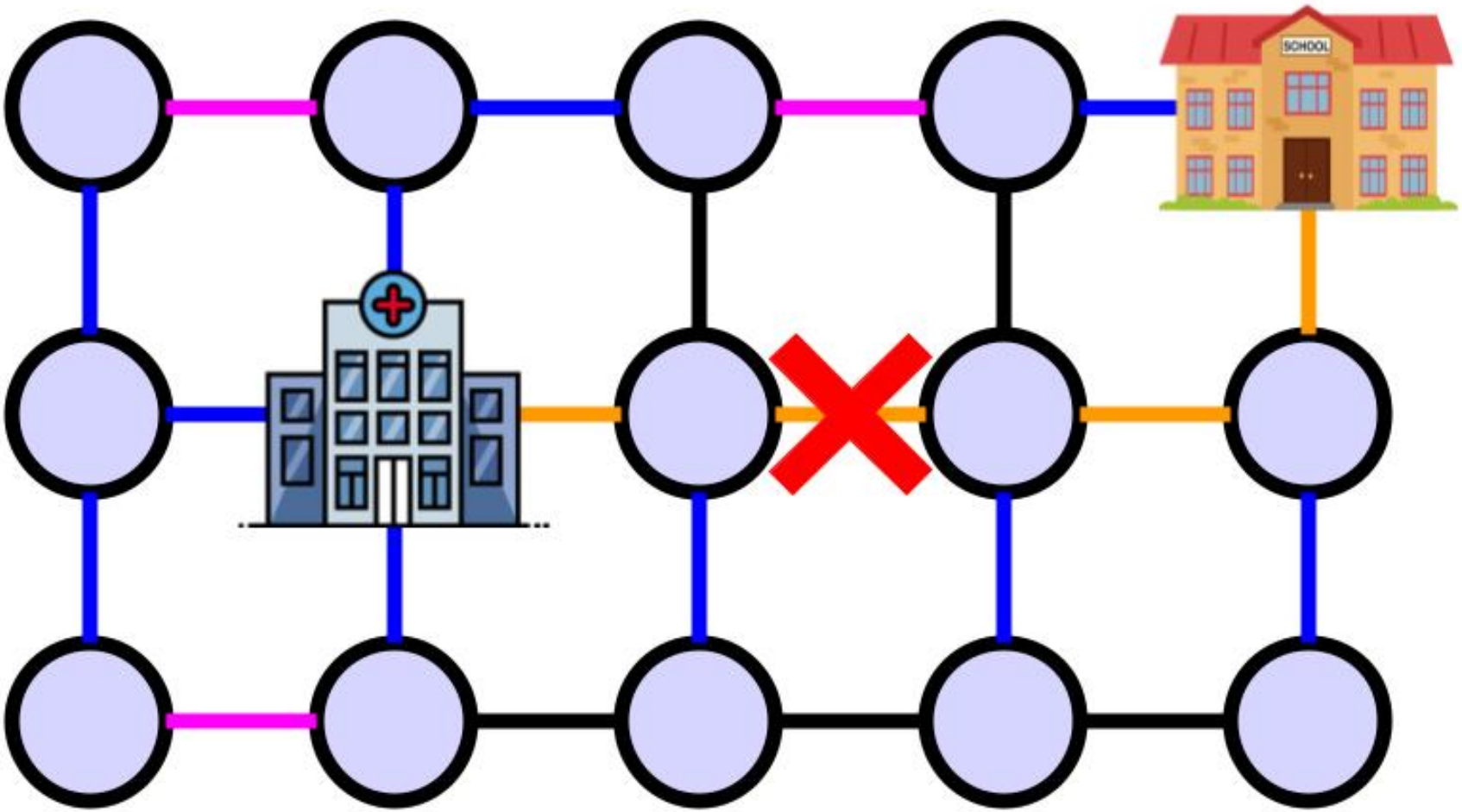
Application Example: Paths



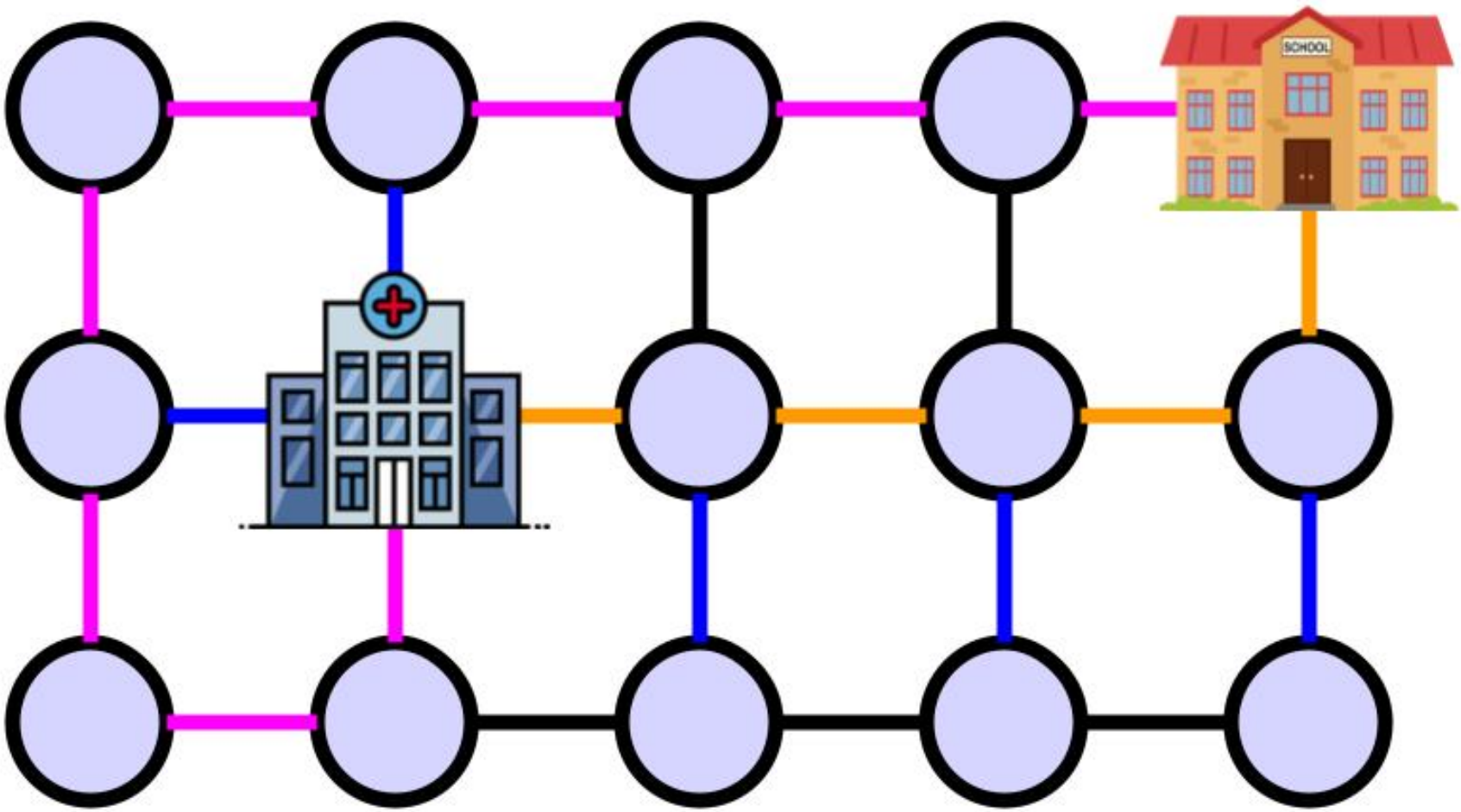
Application Example: Paths



Application Example: Paths



Application Example: Paths



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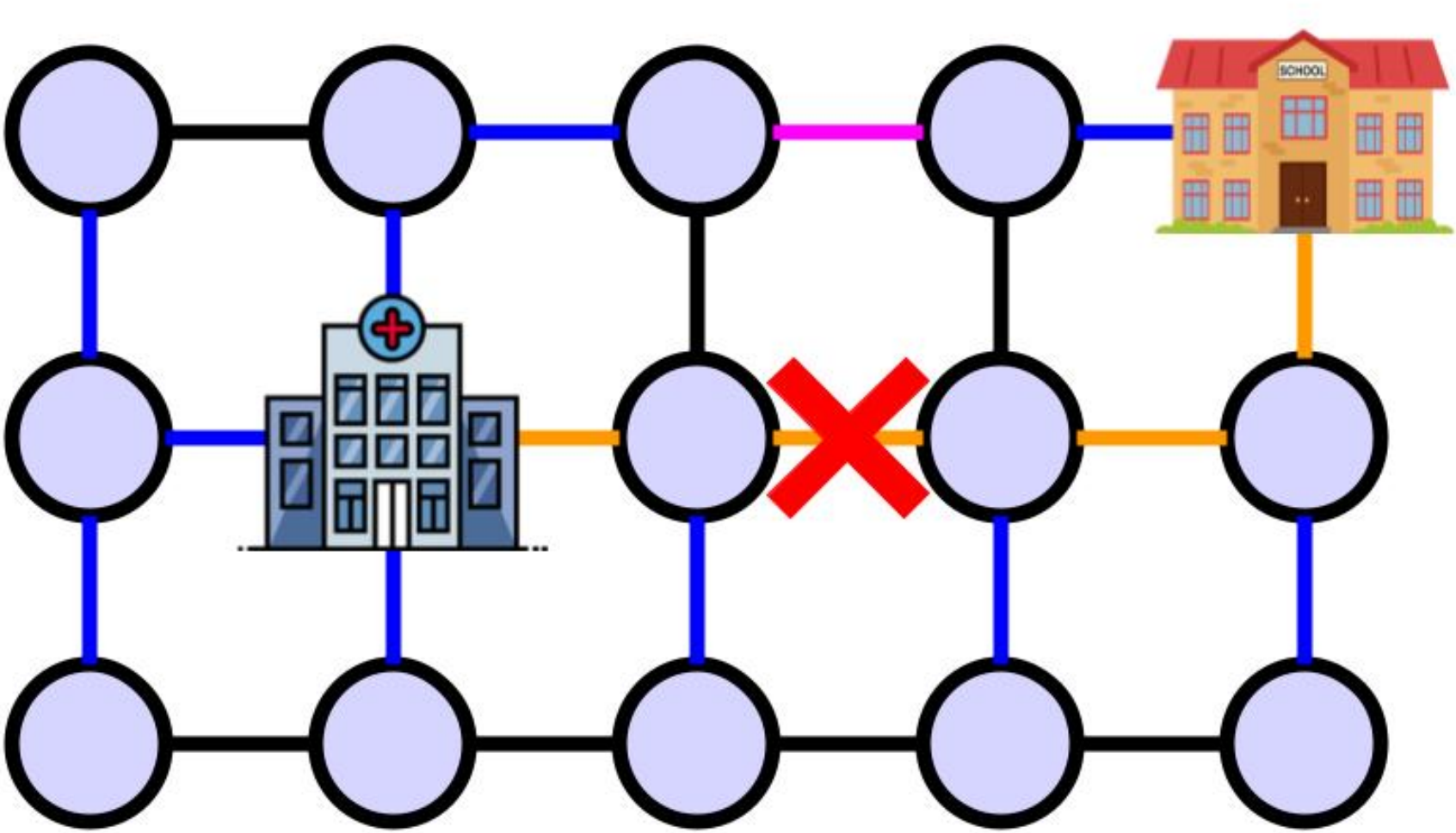

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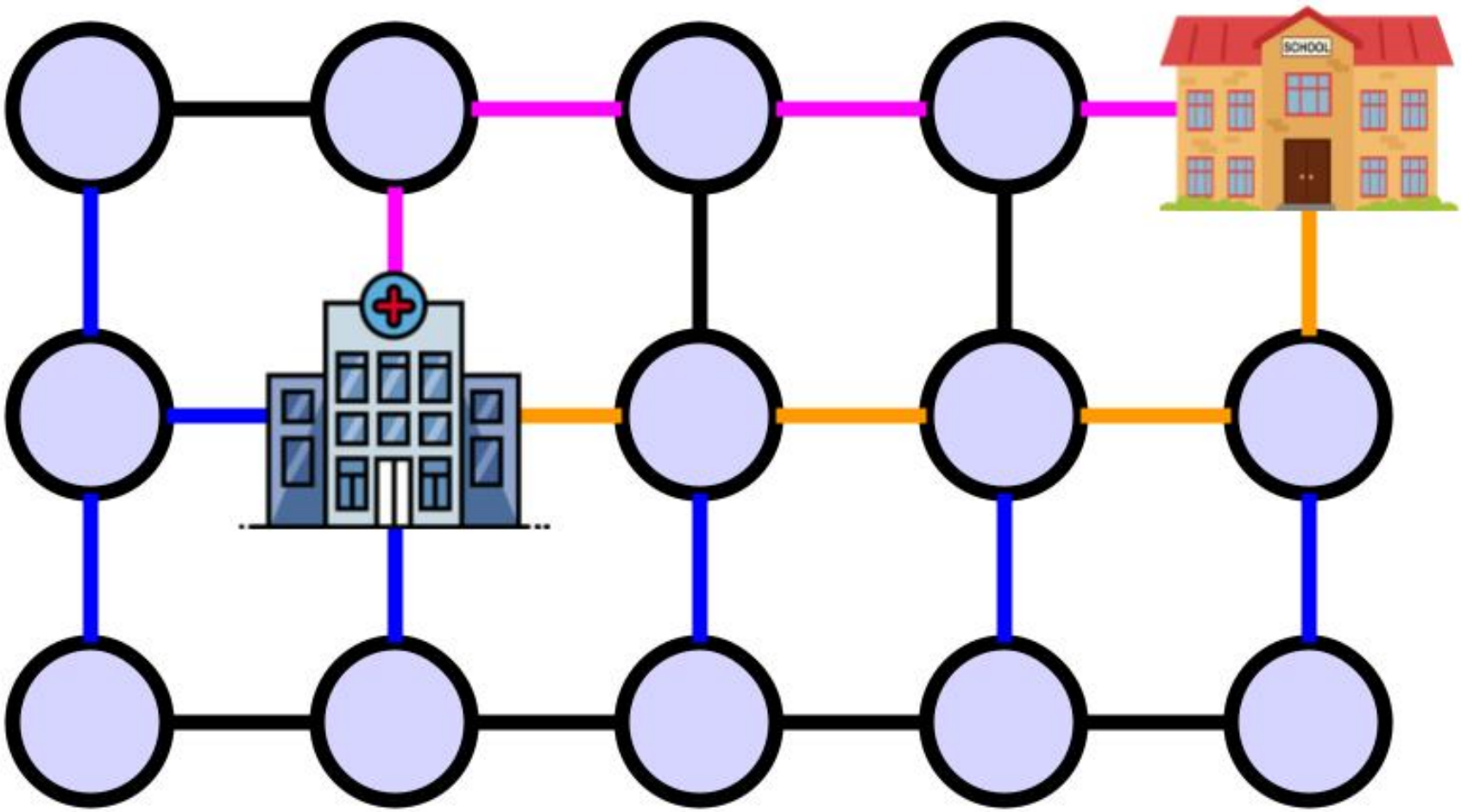
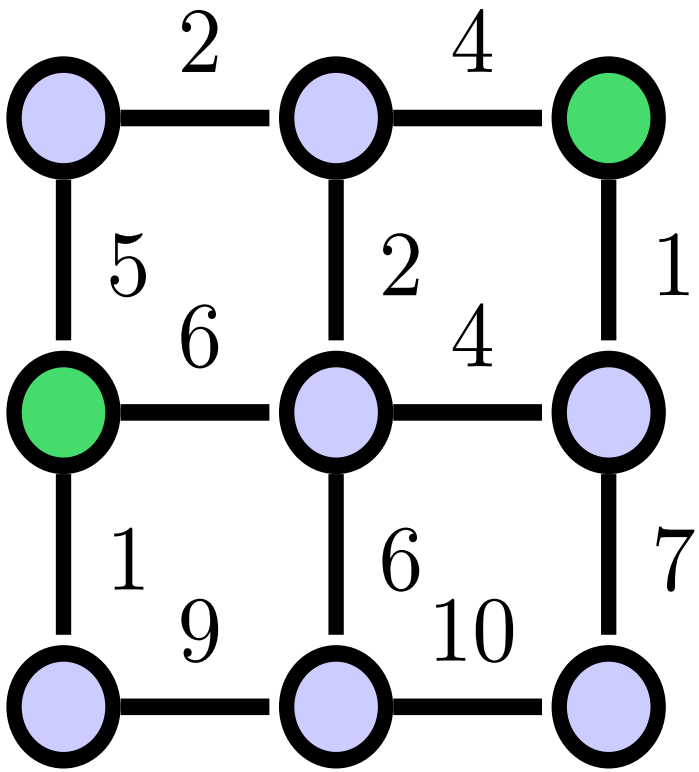
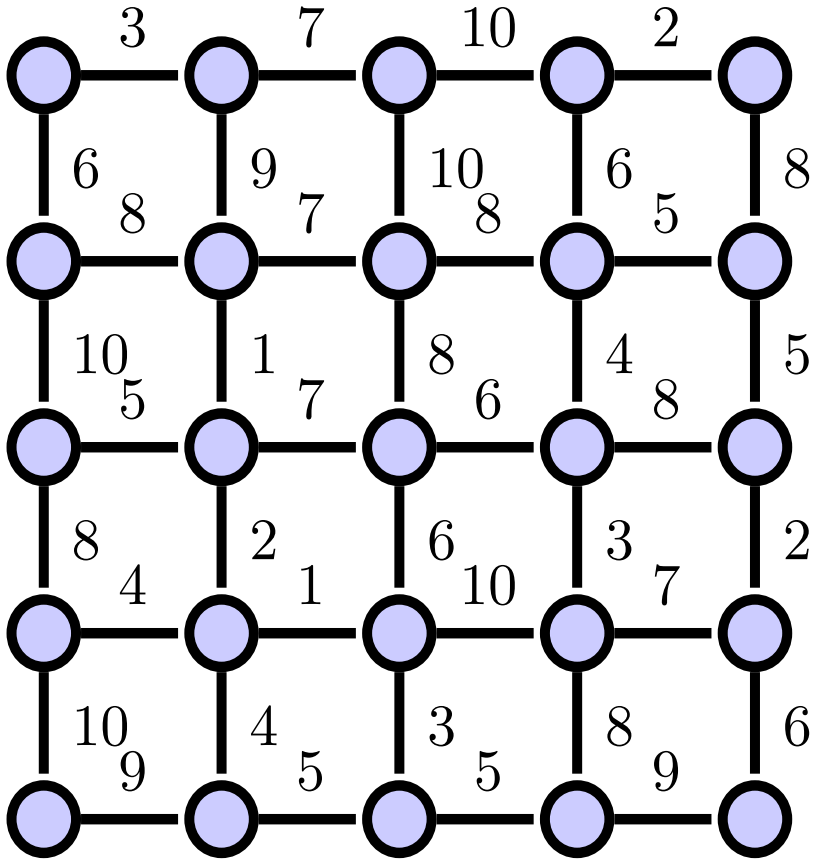


Quick Note: Required Nodes



Outline

MST-First Approach



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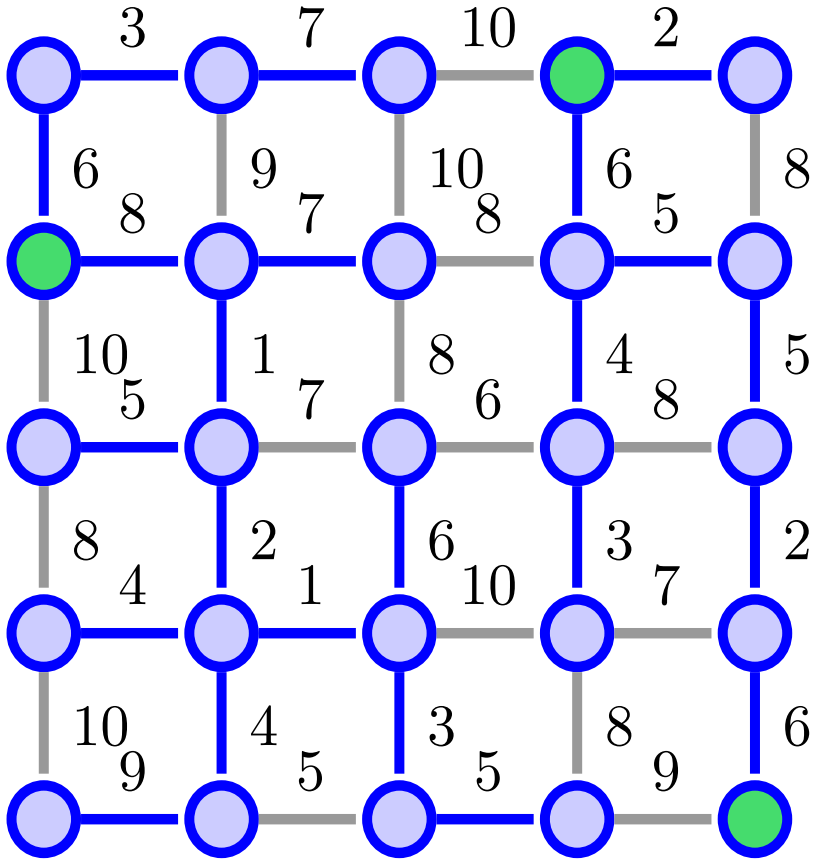

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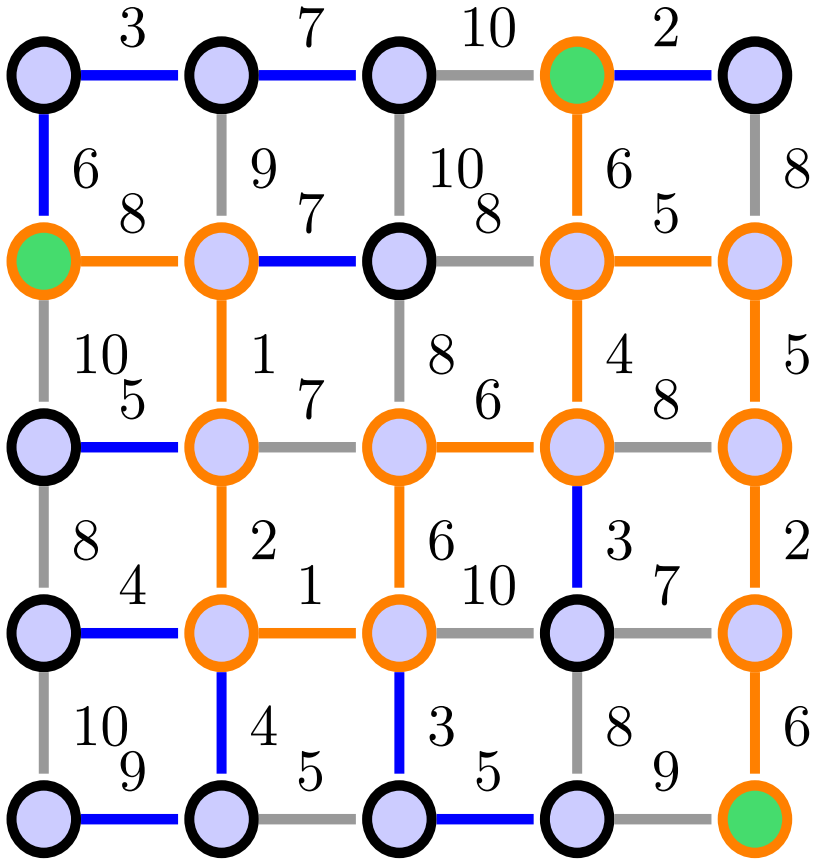

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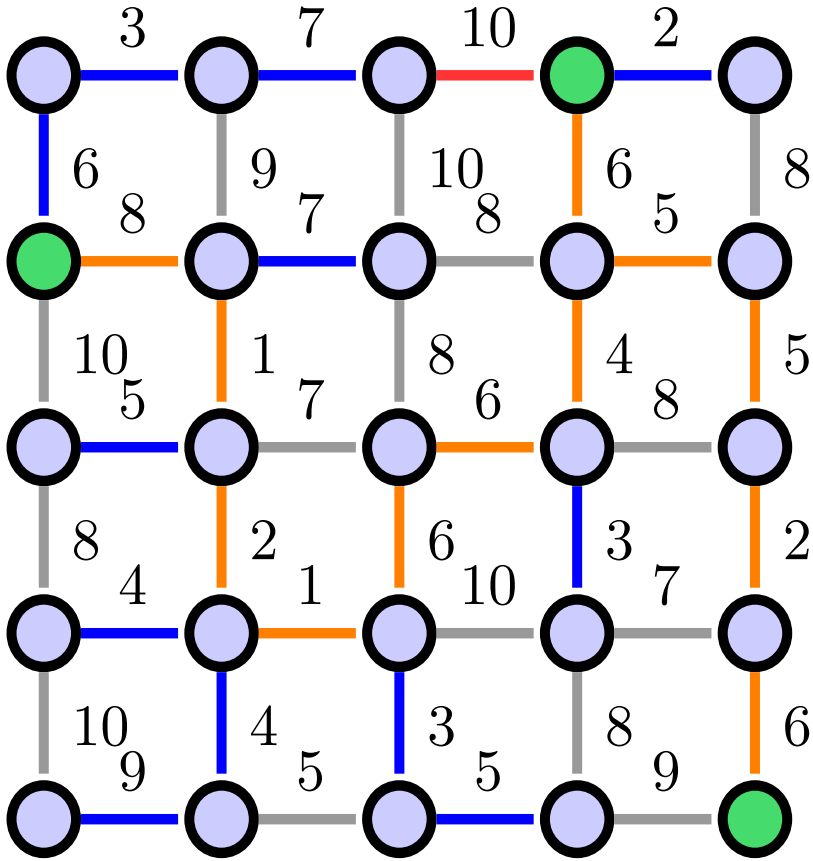

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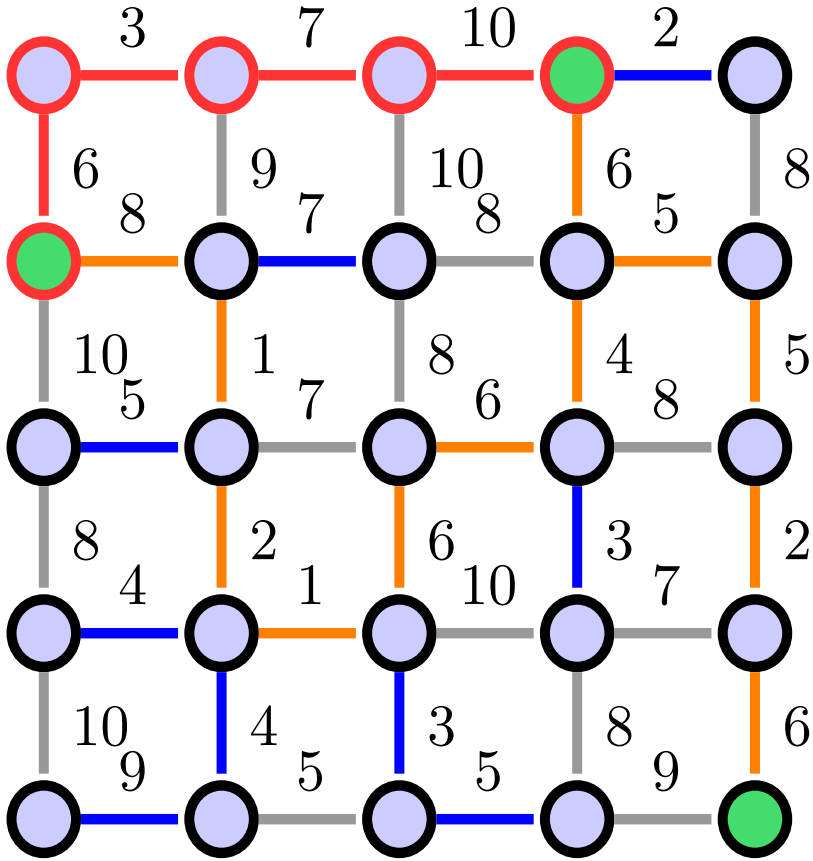

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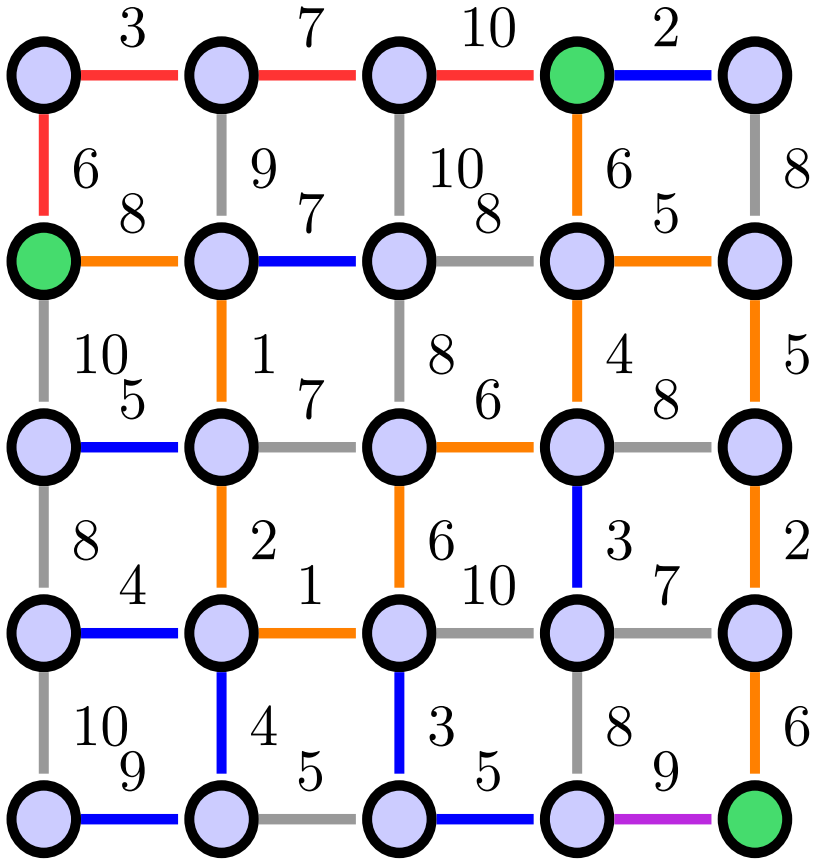

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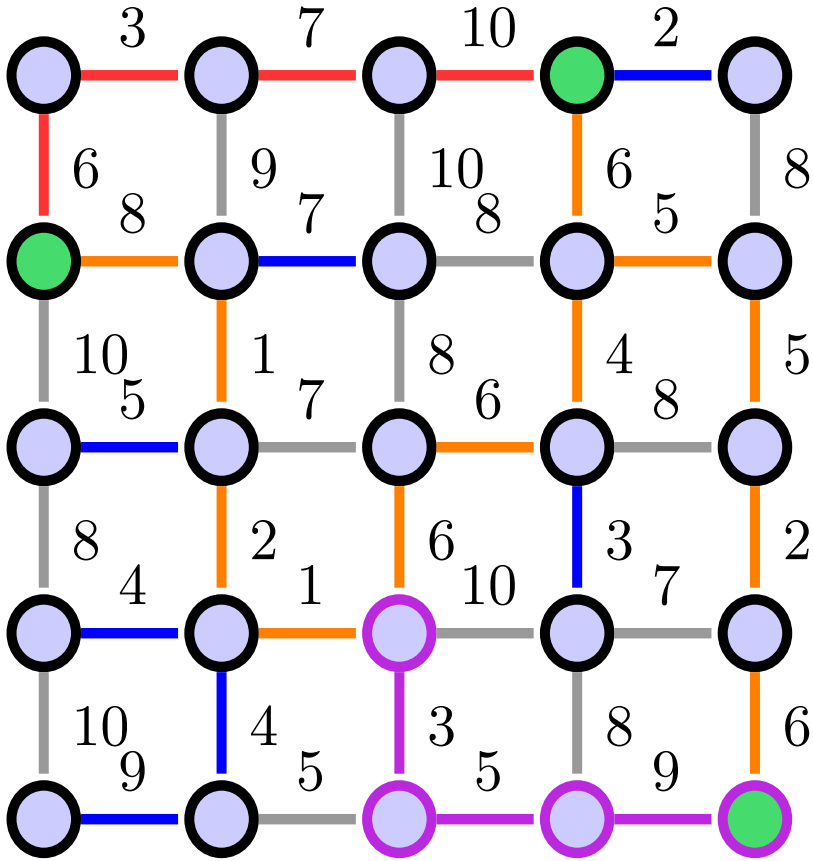

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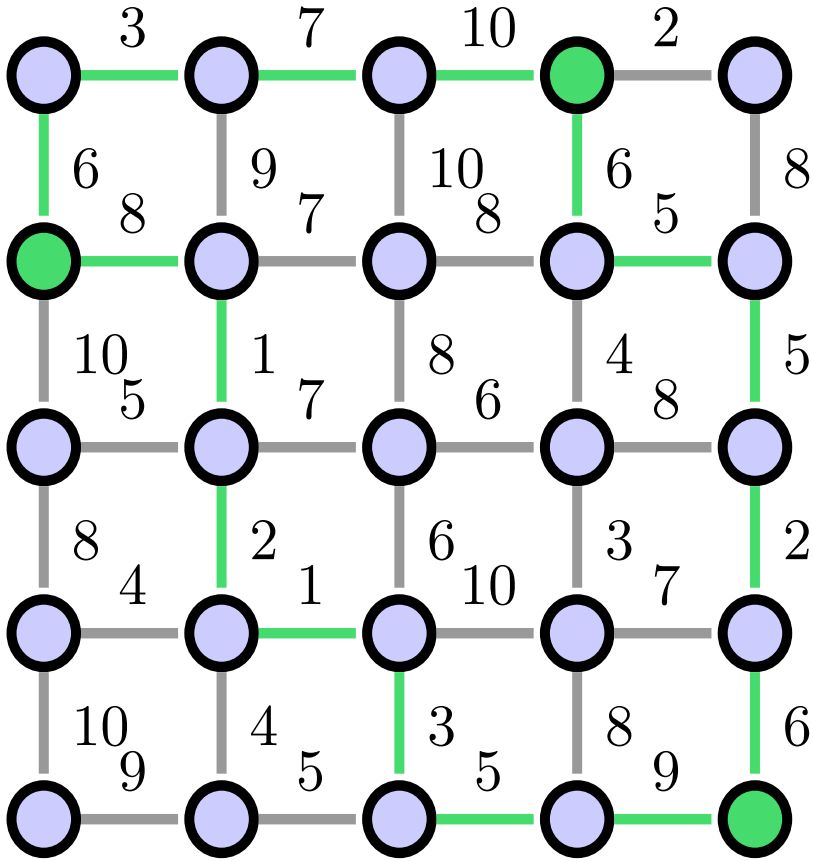

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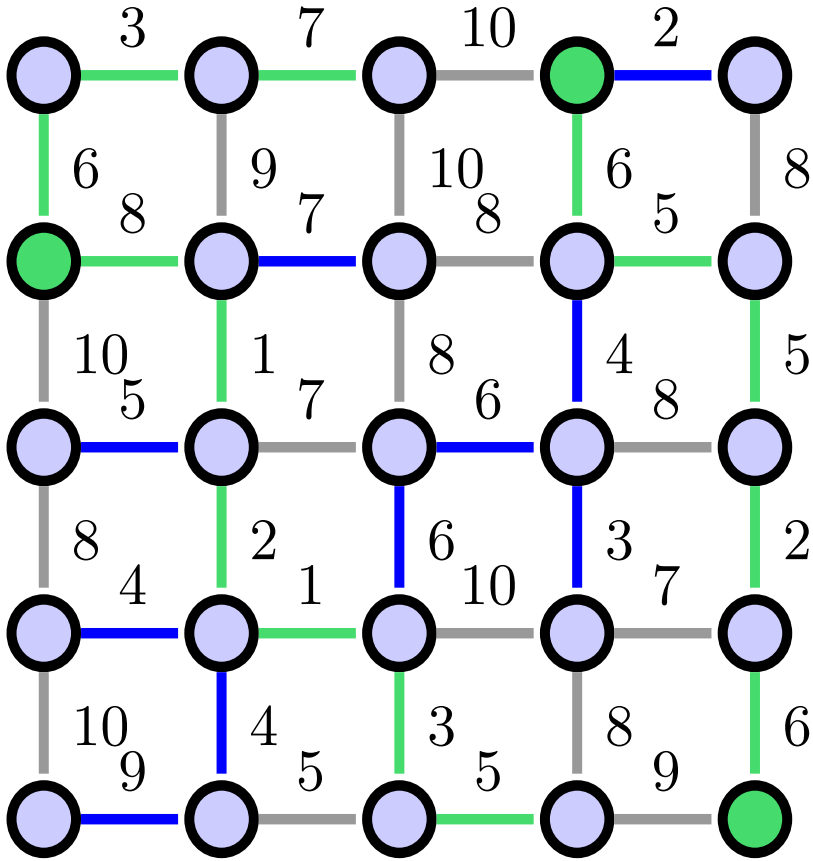

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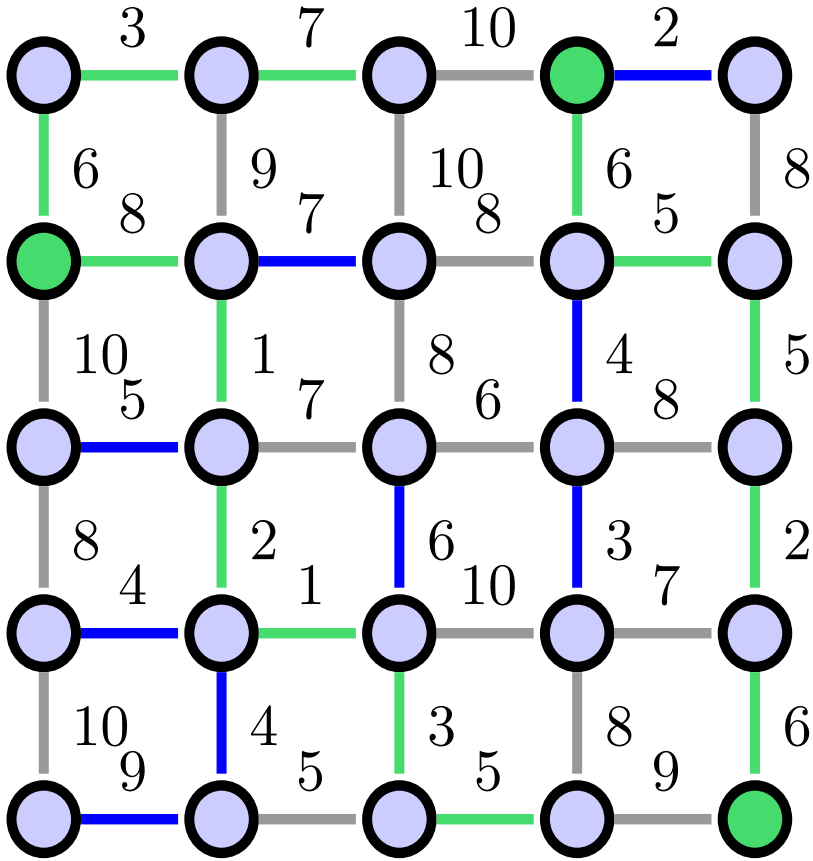
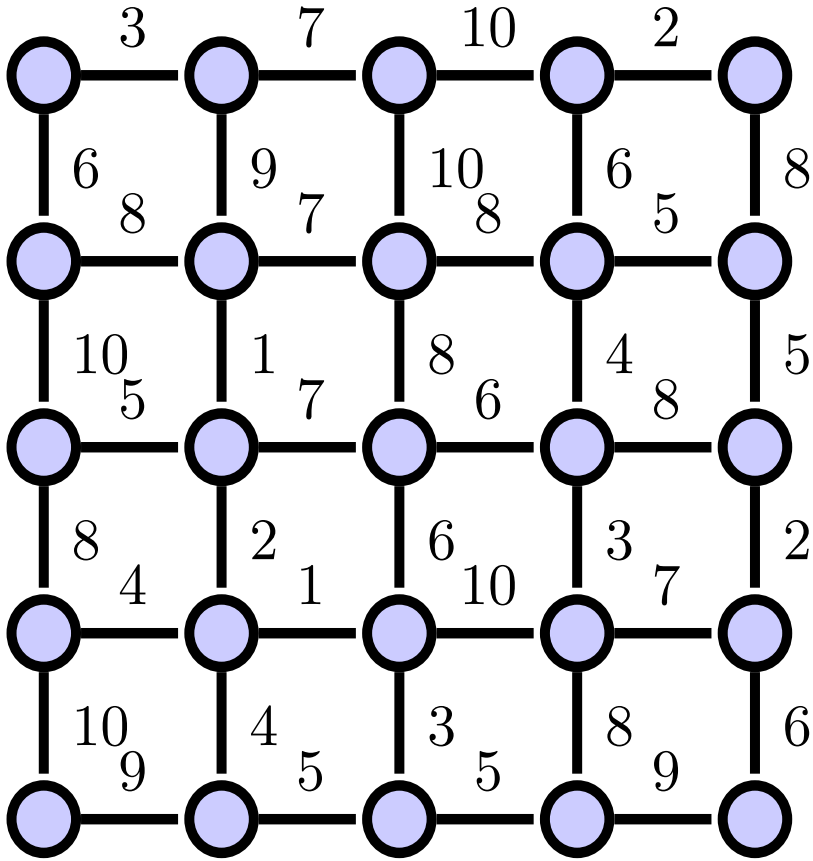


Path-First Approach



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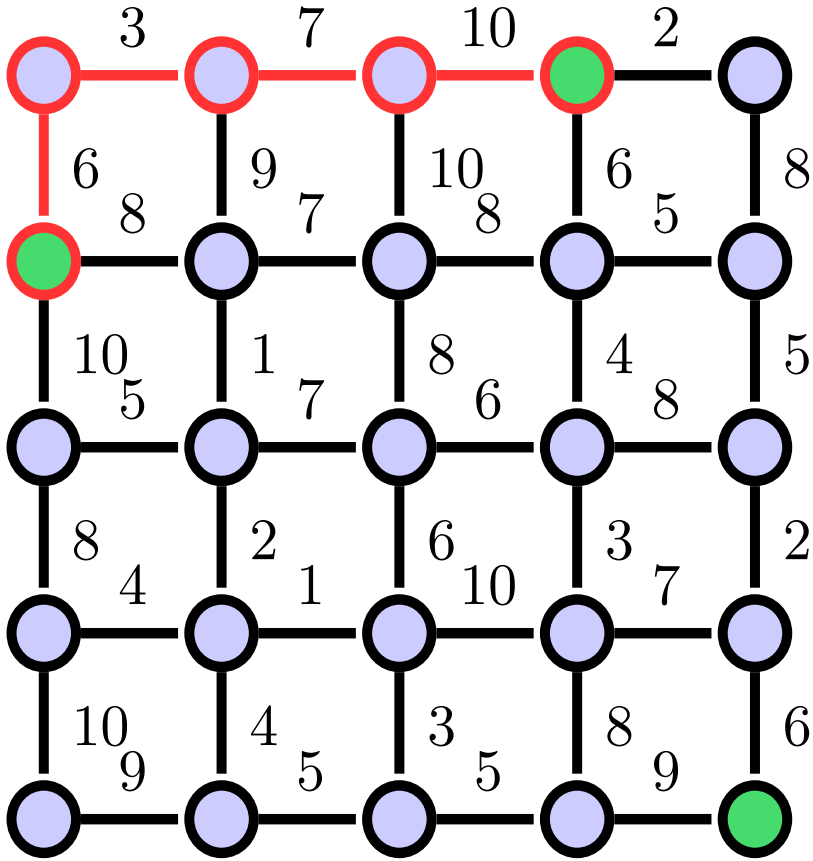

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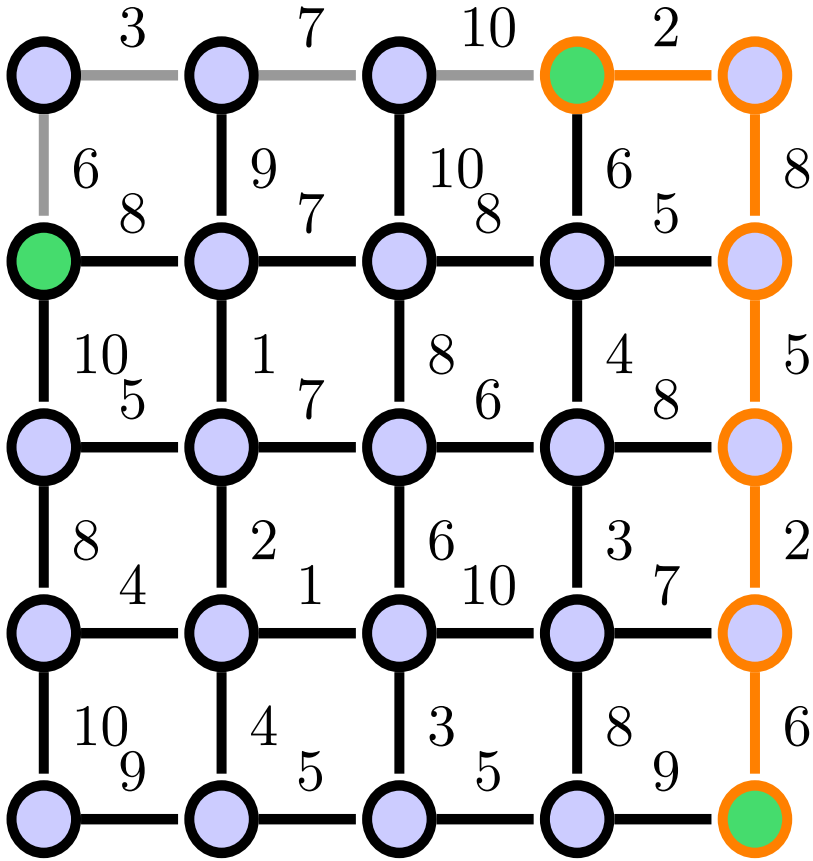

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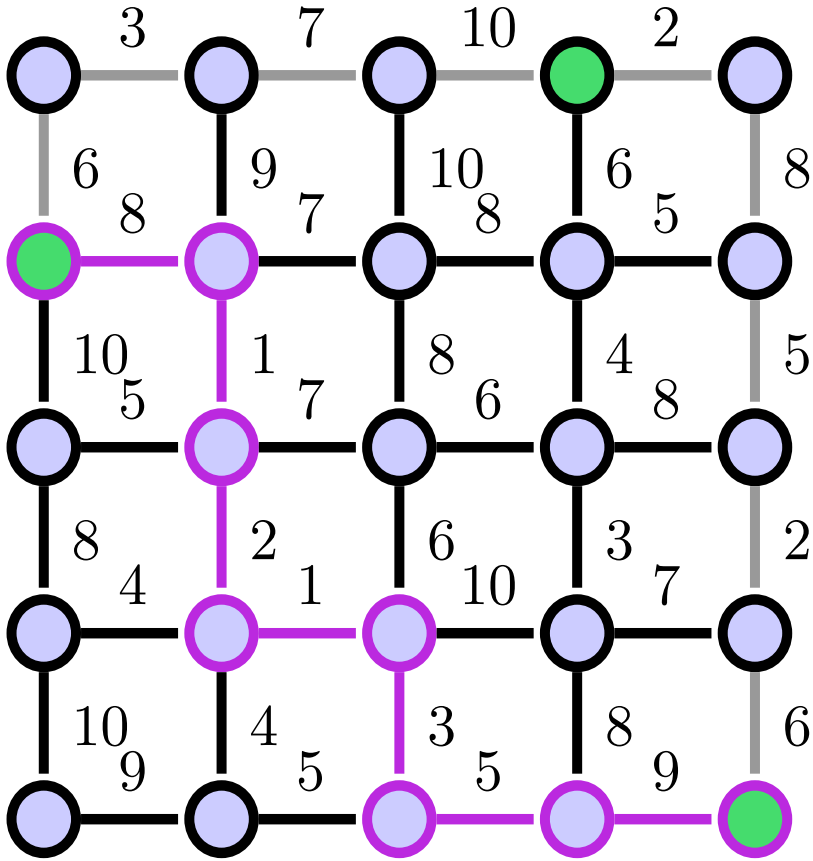

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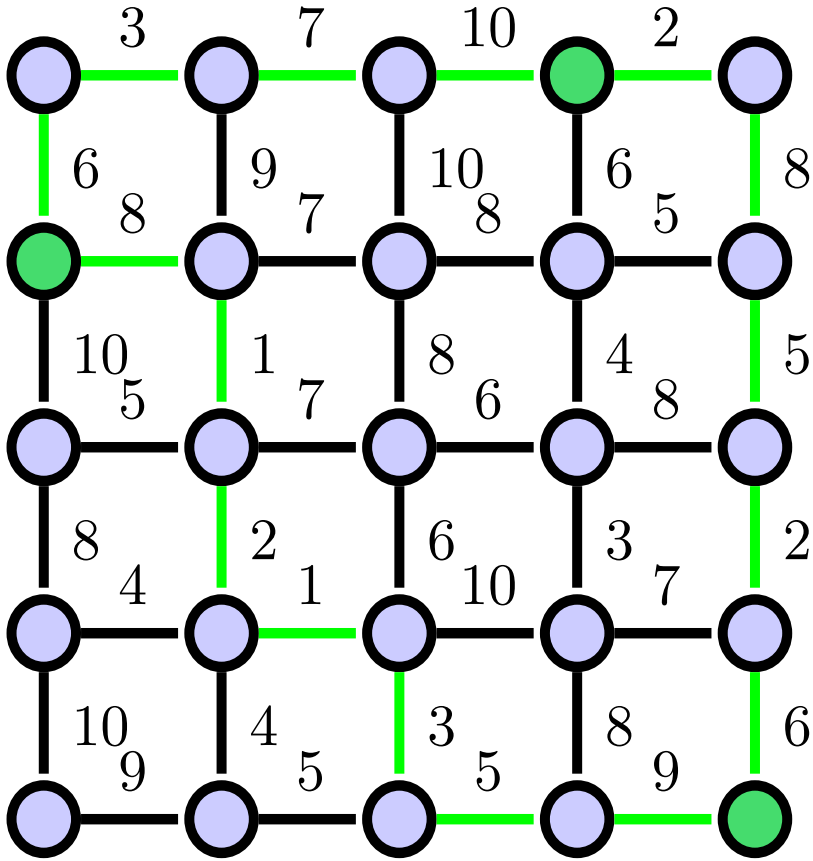

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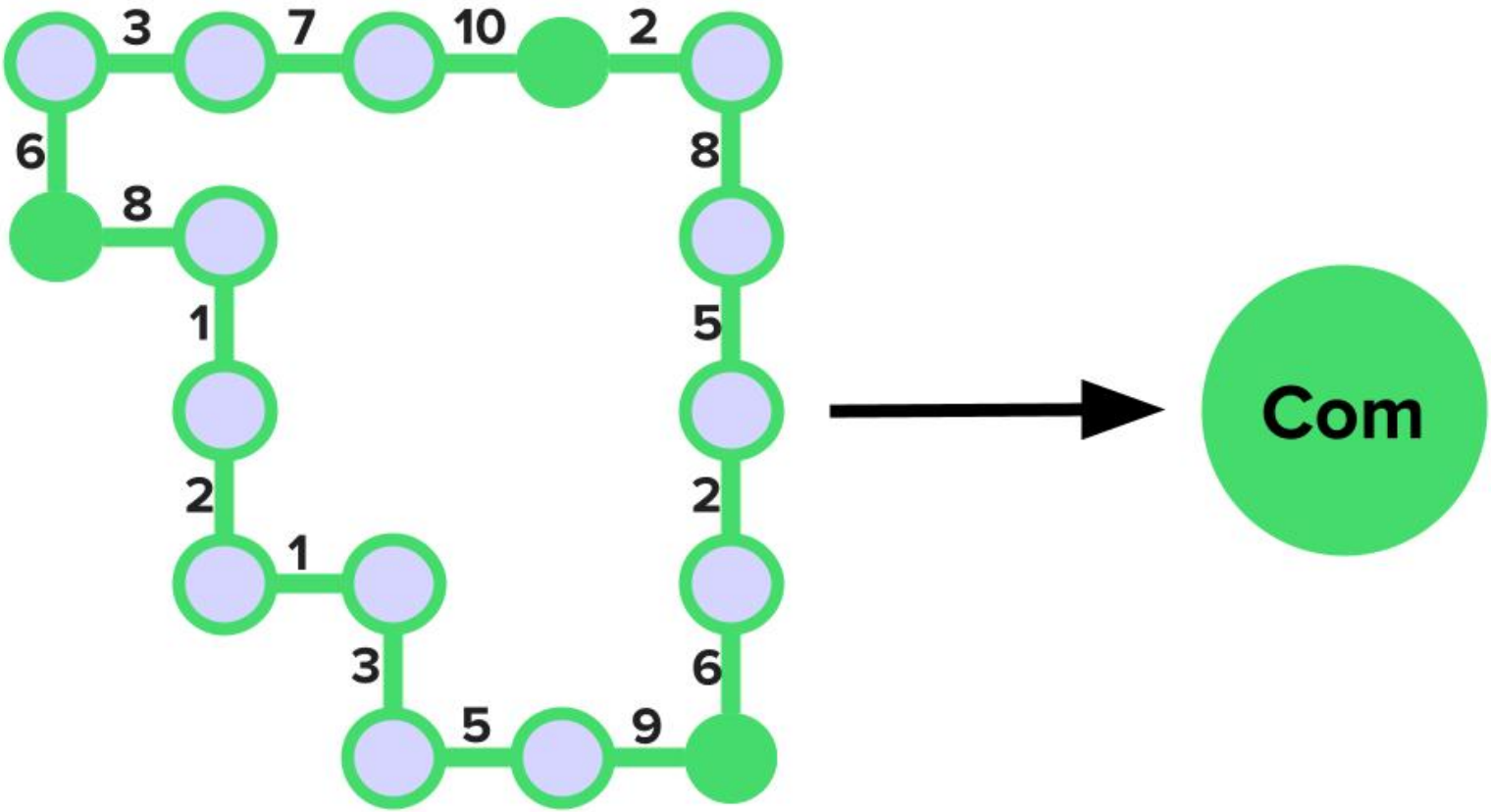

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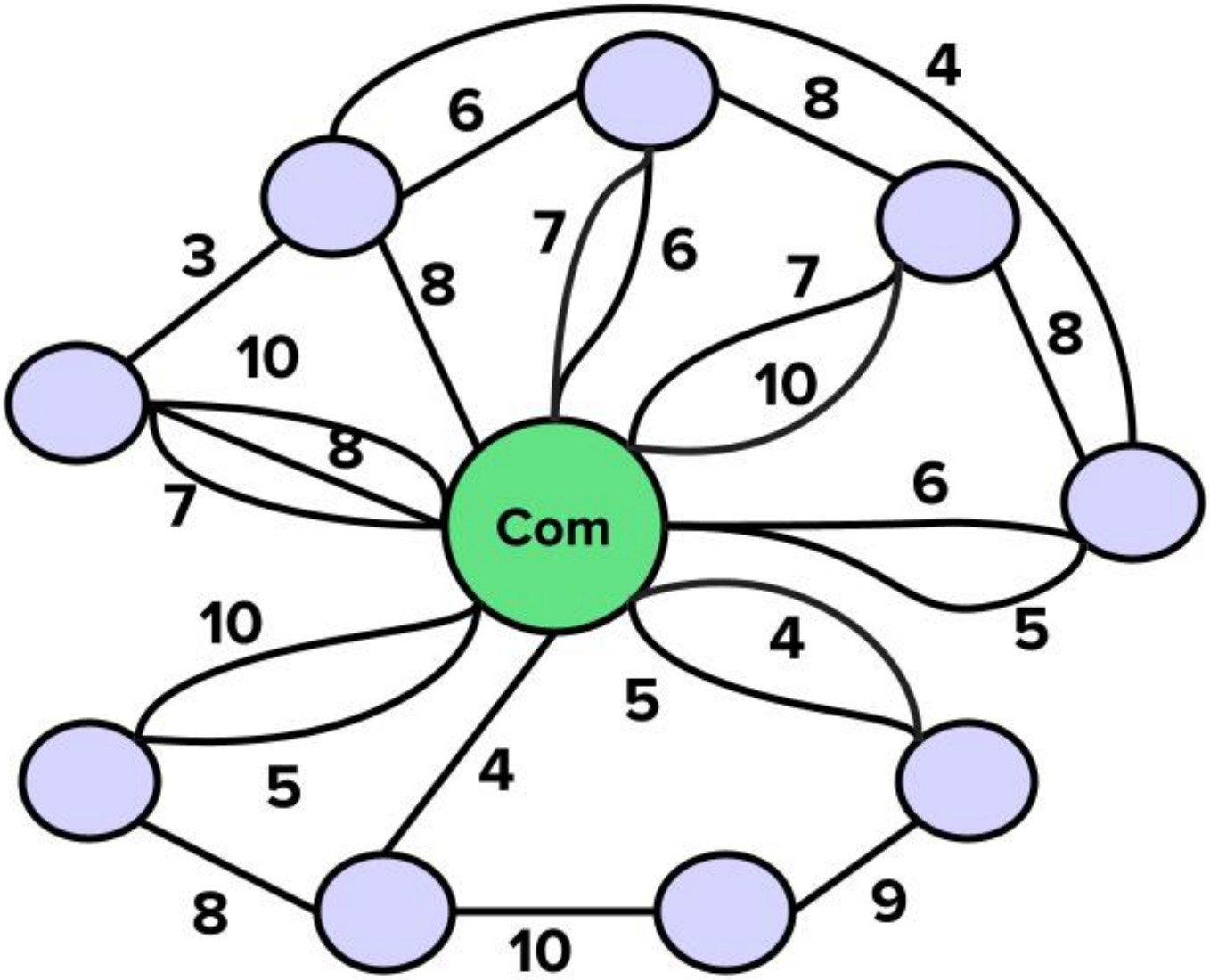

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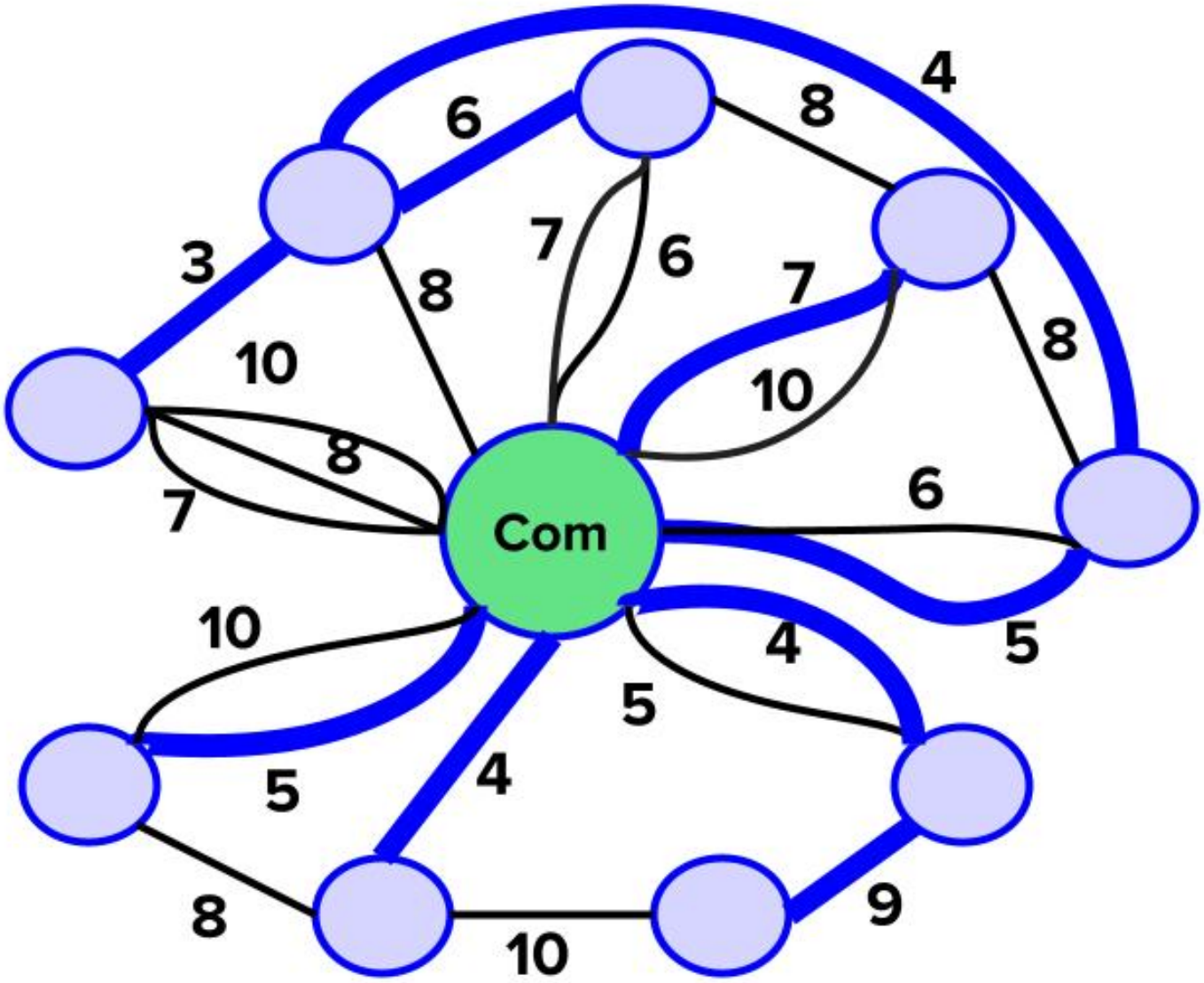

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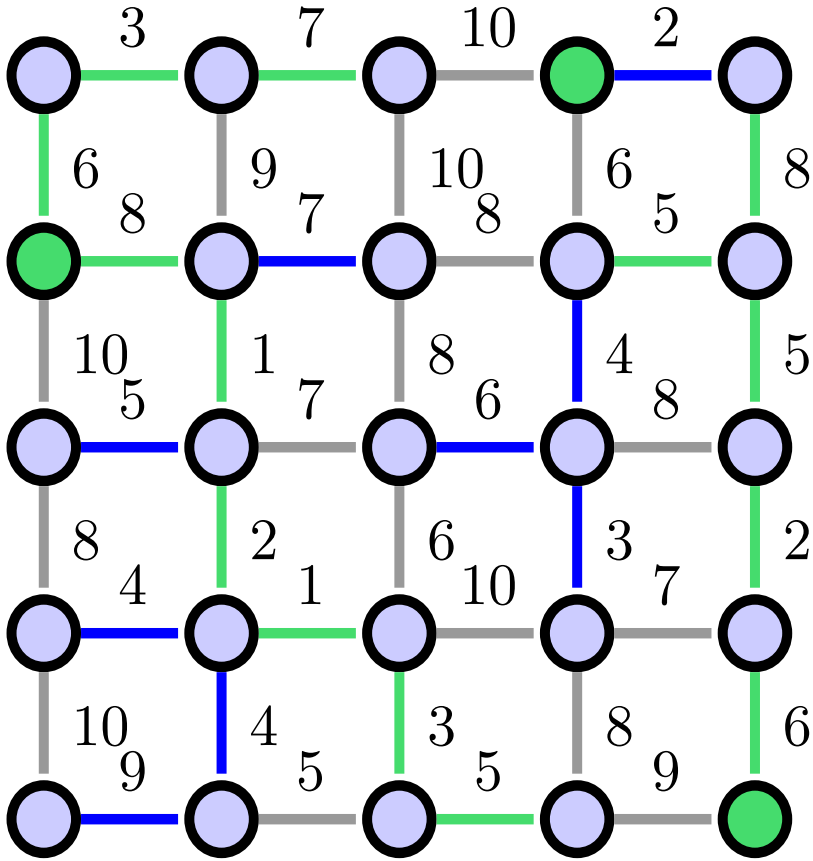
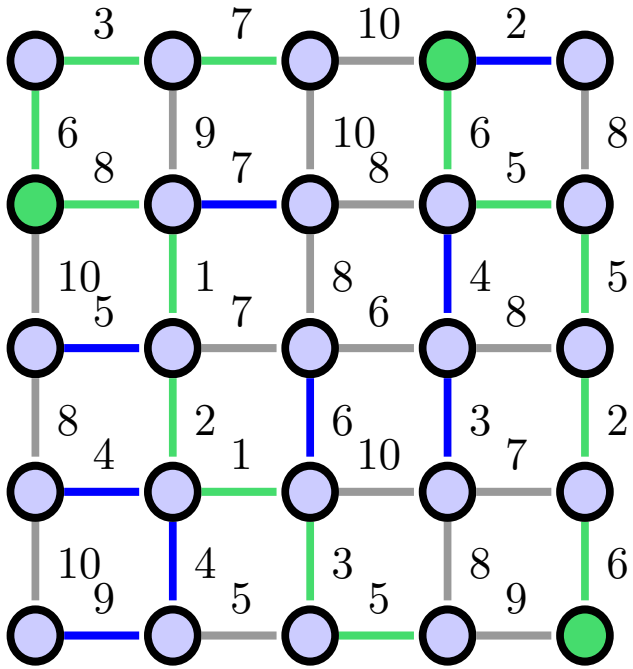


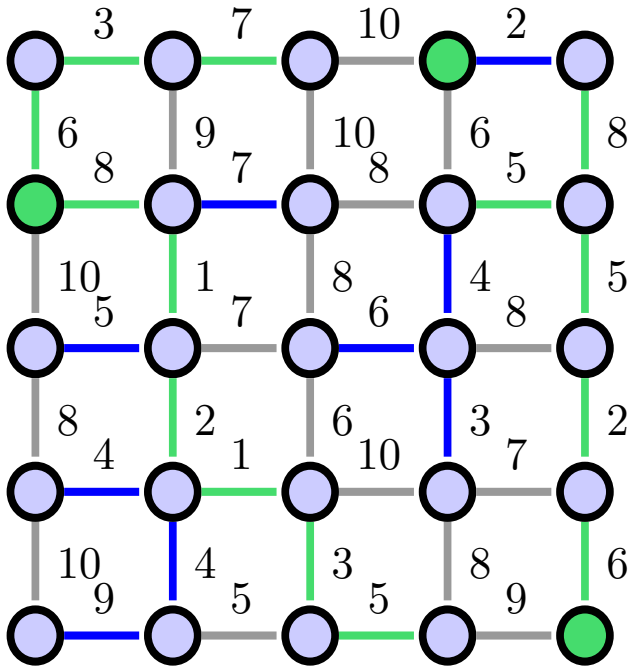
Outline

MST-First Results



MST-First Safety Net
Original Weight: 245
Final Weight: 123

Path-First Results



Path-First Safety Net
Original Weight: 245
Final Weight: 125

Outline

Current and Future Research

- Reducing time complexity
- Continuing to compare the two approaches
- Making various adjustments to the algorithms
- Generalizing beyond grid graphs
- etc.

Thank you!
Any questions?

References

- GeeksforGeeks. (2022, November 17). Applications of Minimum Spanning Tree Problem. https://www.geeksforgeeks.org/applications-of-minimum-spanning-tree/
- R. C. Prim, "Shortest connection networks and some generalizations," in The Bell System Technical Journal, vol. 36, no. 6, pp. 1389-1401, Nov. 1957, doi: 10.1002/j.1538-7305.1957.tb01515.x.

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