

**The Thirteenth Annual  
Nebraska Conference  
for Undergraduate Women  
in Mathematics**

**January 28 - January 30, 2011**

**TALK ABSTRACTS**

## PLENARY TALKS

**Dr. Fan Chung**  
**Professor of Mathematics**  
**University of California, San Diego**  
**Graph Theory for the Information Age**

Nowadays we are surrounded by numerous large information networks, such as the WWW graph, the telephone graph and various social networks. Many new questions arise. How are these graphs formed? What are basic structures of such large networks? How do they evolve? What are the underlying principles that dictate their behavior? How are subgraphs related to the large host graph? What are the main graph invariants that capture the myriad properties of such large sparse graphs and subgraphs? In this talk, we discuss some recent developments in the study of large sparse graphs and speculate about future directions in graph theory.

**Dr. Linda R. Petzold**  
**Professor of Mathematics**  
**Cornell University**  
**Modeling and Analysis of Circadian Clocks**

Circadian clocks govern daily behaviors of organisms in all kingdoms of life. In mammals, the master clock resides in the suprachiasmatic nucleus (SCN) of the hypothalamus. It is composed of thousands of neurons, each of which contains a sloppy oscillator: a molecular clock governed by a transcriptional feedback network. Via intercellular signaling, the cell population synchronizes spontaneously, forming a coherent oscillation. This multi-oscillator is then entrained to its environment by the daily light/dark cycle. We use simulation and analysis to elucidate the mechanisms of intercellular signaling and light entrainment.

## Talks by Undergraduate Students

**Darlayne Addabbo, Rutgers University**

**Noncommutative Partial Fractions and Continued Fractions**

My presentation will involve an overview of two results from my research project this past summer. The first result is a generalization of the method of partial fraction decomposition to non-commutative cases. I showed that, if a partial fraction decomposition exists for a given polynomial fraction, the coefficients of the decomposition can be expressed by the solutions to a specific matrix equation. The second part of my presentation will involve a discussion of a generalization to the non-commutative case of Galois' result that every periodic continued fraction is the solution to a quadratic equation.

**Holly Arrowood, Furman University**

**$L(3, 2, 1)$ - Labeling of Three-Dimensional Grids**

Vertex labeling has been a widely studied concept in the field of graph theory. The type of labeling studied in this project is called  $L(3, 2, 1)$ -labeling, and it is related to distances between vertices in the graph. In my project, I determine optimal  $L(3, 2, 1)$ -labelings for three-dimensional grids, including the infinite case.

**Stephanie Bobo, Belmont University**

**Scheduling Prison Guards and Faculty: Examples Using Linear Programming**

The talk will present the solutions of scheduling problems using linear programming to find an optimal solution. In particular, we look at a scheduling problem dealing with prison guards in which the objective is to minimize the number of overtime hours; this problem is based on a UMAP module published by COMAP. We re-implemented a solution of this problem in MATLAB. In addition, we look at an unsolved scheduling problem: assigning faculty to classes so that the workload is fairly distributed. We solve this problem with real data from Belmonts math and computer science department as an example and compare to past schedules.

**Natasha Bollers, City University of New York Medgar Evers College**

**Mathematical Model for One Dimensional Quasi- Crystals**

This project seeks to investigate methods of modeling mathematically a one-dimensional quasi-crystal. We began the project by understanding the Crystallographic Theorem that describes the permissible types of rotational symmetries of a crystal. A quasi-crystal is defined as a crystal like solid with "forbidden" symmetries. We can model both a one dimensional crystal and a one dimensional quasi-crystal using the Cut and Project Method along a line of a rational or an irrational slope respectively. All this work will allow us to describe quasi-crystals abstractly, as infinite words over an alphabet with two letters whose succession follows a certain pattern. We will continue to study the physical properties of this mathematical model using mostly combinatorial arguments combined with linear algebra and careful pattern recognition.

**Laura Booton, Nebraska Wesleyan University**  
**Kristen May, Canisius College**  
**Algorithms for Automated Spoken Word Recognition**

We present an algorithm that mathematically recognizes spoken words by using wavelet decomposition. The distinguishing characteristics of the words are compared using the level 1 detail coefficients. We develop a diagnostic algorithm in order to compare the accuracy of word identification among different wavelet bases. We implement a statistical study to determine the best wavelet to decompose the signals of spoken words.

**Mary Bottoms, North Georgia College and State University**  
**Student Engagement**

The National Survey of Student Engagement (NSSE) measures key engagement characteristics for undergraduate students. Various critiques to the original NSSE development exist in the literature including Exploratory Factor Analysis (EFA) and Confirmatory Factor Analysis (CFA) approaches. Some researchers have attached student performance data to determine if the key NSSE indicators impact student success measures. An undergraduate research group at North Georgia ran EFA on the unattached questions. Using a maximum likelihood approach ( $\delta = .4$ ) and direct obliteration, we were able to look at five additional explanatory variables. We found no detailed reports of impact of the NSSE key indicators on first college generation students in the literature. Our research group's multivariate analyses included multiple regression techniques, logistical regression and ANOVA procedures. The presentation summarizes the key findings for institutions like ours that serve a large proportion of first generation college attendees.

**Brittany Bunker, University of Nebraska-Lincoln**  
**Population Dynamics of Gregarine Parasites in Damselfly Hosts**

Damselflies play host to a rich community of parasites, including protistan parasites known as gregarines. Observations of gregarine parasites within both larval and adult damselfly hosts throughout the course of a summer give insight to how the population distributions change over time and at different locations. Gregarine populations were found to fit the negative binomial distribution, in which most of the hosts have zero or few parasites, while fewer hosts have large numbers of parasites. The negative binomial distribution is indicative of uneven dispersal, and presents interesting mathematical and biological implications. There has been substantial interest in macroparasite infections of humans and livestock; however, much is still unknown in quantifying many aspects of macroparasitic life cycles. A study of macroparasitic infections of insects may contribute to the overall understanding of parasite fitness and behavior. This work was supported by NSF Research for Undergraduates in Theoretical Ecology.

**Christine Caples, Fairfield University**  
**Geometric 2-D Shape Modeling**

In order to perform automated shape recognition, a computer must be given a representation of each shape. One such representation is the medial representation, which can be thought of as the skeleton of the shape. This project compares different medial representations based on optimality, which is determined by computing the minimum the number of bits required for the representation of a particular shape. Specifically, we explore variants of the Blum Medial Axis in order to obtain the most optimal medial representation for an ellipse and a class of near-ellipses.

**Emma Chiappetta, Bowdoin College**  
**Unifying Geometry: The Cayley-Klein Scheme**

In the 1870s Felix Klein sought a unified approach to the range of new geometries discovered earlier in the century. One result, building on earlier work of Arthur Cayley, was the emergence of what is known today as Cayley-Klein geometries, all subgeometries of projective geometry. The unifying aspects of Cayley-Klein geometries are the importance of conics and Cayley's generalization of distance in projective space. Interestingly, this formula is the same for both angle and distance across all of the Cayley-Klein geometries, differing only up to a choice of constant. This constant can either be real, imaginary or infinite. The constant, and thus the type of geometry, is determined by the original conic. As established by D. M. Y. Sommerville in 1910, there are only nine possible Cayley-Klein geometries in two dimensions, including the usual Euclidean geometry, hyperbolic geometry, and elliptic geometry. Several of the other geometries serve as physical models of space-time.

**Natalie Coston, Northern Arizona University**  
**The geometry of one-relator groups**

In the field of abstract algebra, a one-relator group is given by a set of generators and one relation defining how the generators interact to form the group. These one relator groups have properties that can be further explored, one of them being the concept of  $a$ - $T$ -menability. A group  $G$  is called  $a$ - $T$ -menable if it admits an affine, isometric, and metrically proper action on a Hilbert space. This presentation explores geometric constructions for few concrete  $a$ - $T$ -menable groups and addresses the possibility of applying such constructions to one-relator groups.

**Meredith Daw, Northern Arizona University**  
**Karin Kralicek, Northern Arizona University**  
**A New Family of Matroids**

The arrangement of hyperplanes in the  $n$ th dimension Complex plane can often be expressed as a matroid. Our question is one of finding a way to generalize a single type of matroid in lower dimensions to higher dimensions. We have found one new family of such matroids that may generalize further.

**Helene Duke, Providence College**  
**Edge-Enhancing Speckle Denoising for Ultrasound Images**

Ultrasound images contain pervasive granularity which interferes with human analysis and automated processing of the images. Towards removing this granularity we develop an edge-enhancing denoising model which treats the granules in ultrasound images as speckle noise. Our partial differential equation based model, derived from a minimizing functional constrained by a new noise equation, is more efficient in removing speckle noise in ultrasound images than the existing models. A corresponding explicit time-stepping scheme from a non-standard finite difference discretization is proven to be stable. We also discuss and evaluate some parameterization techniques. Our scheme compares favorably with existing techniques, providing smooth results very quickly. Images clarified by our scheme are shown to be much more suitable for further automated processing tasks.

**Samantha Erwin, Murray State University**

**Using matrix analysis to model the spread of an invasive plant, *Alternanthera philoxeroides***

*Alternanthera philoxeroides*, more commonly known as alligator weed, is an invasive species indigenous to South America. With its alarming invasion of south east United States water ways, understanding its invasiveness of this plant species is both important and imperative. Utilizing experimental growth data obtained over the summer of 2010 matrix analysis is used to precisely model the growth of alligator weed. These matrices are population projection models whose eigenvalues represent the growth rate of alligator weed in its different stages of the life cycle. A high growth rate is a key feature of successful invaders. Residuals and sensitivity analysis are being performed to test the accuracy and importance of the models.

**Brooke Fox, Northern Arizona University**

**Maggie Schroeder, Northern Arizona University**

**Recent Developments in Numerical Semigroups**

A numerical semigroup is a subset of the positive integers that contains zero, is closed under addition, and has finite complement. They are useful for exploring certain problems in commutative algebra. We will present recent developments and open questions/problems in numerical semigroups. No prior knowledge of abstract algebra is needed.

**Jessica Fuller, Seton Hall University**

**An Elementary Approach to Laplacian Integral Non-Threshold Split Graphs**

A graph  $G$  is a split graph if its node set can be partitioned into a clique and an independent set. Threshold graphs are split graphs with the added property that for all pairs of nodes  $u$  and  $v$  in  $G$ ,  $N(u) - \{v\}$  is a subset of  $N(v) - \{u\}$  whenever  $\deg(v)$  is less than or equal to  $\deg(u)$ . The number of spanning trees for such graphs are computable using the Laplacian matrix for the graph, calculating its eigenvalues, and applying a corollary to Kirchhoffs well-known Matrix-Tree Theorem. These eigenvalues are always integers in the case of threshold graphs, but until recently it was unknown whether any non-threshold split graphs were Laplacian integral. We present a conjecture for the eigenvalues of a class of split graphs that we call  $q$ -IPS ( $q$ -Ideal Proper Split) and can indicate specific graphs that are Laplacian integral. This work is independent of Kirkland et. al., who recently discovered the same Laplacian integral split graphs using balanced incomplete block designs.

**Nicki Gaswick, University of Nebraska-Lincoln**

**Fractional Nabla-Difference Calculus**

We study some foundational properties of difference calculus with respect to the reverse, or nabla, difference operator. We can relate some of our findings to analogous properties which use the forward difference operator, but we found that using the nabla operator highlights several features of difference calculus, such as the importance of domain choice. Beginning by establishing a Fundamental Theorem of Discrete Calculus for the nabla operator, we use this and several other properties of the nabla operator to proceed to establish a generalized Leibniz's Rule, Power Rule, Composition Rules, and Binomial Theorems. Building on these properties, we derive a solution to a general initial value problem, establish a Laplace transformation for nabla difference equations and use it to verify several of the preceding properties.

**Laila Gharzai, University of Nebraska-Lincoln**

**Laser-driven Electron and X-ray Beams for Imaging of Dense Structures Relevant to Biomedical Applications.**

X-rays have been the method of choice for medical imaging since they can penetrate deep into the body and also resolve small structures. The ideal x-ray device would produce photons with energy 100 KeV with high spectral and spatial brightness, a small source size, and be compact and relatively expensive. High-power laser systems offer the possibility of producing such x-ray sources. The Diocles laser system produces electron beams with energy 10-400 MeV over an acceleration distance of 1-4 mm. These electrons can produce x-rays by Thomson scattering or betatron oscillations. These x-rays are extremely bright (10<sup>9</sup> photons/sec at 10-500 keV) and a source size of a few microns. Both electrons and x-rays from this source can be used for imaging purposes. The requirements for effective imaging, and in particular the requirements to obtain the highest possible contrast and resolution for specific configurations were studied. Imaging measurements were performed with calibrated image plates and dose requirements for optimal imaging were determined. This is joint work with S. Banerjee and D. Umstadter at the University of Nebraska-Lincoln.

**Amanda Goodrick, Slippery Rock University**

**Modeling the Motion of the Robot Arm of a Space Shuttle Subject to a Torque Constraint**

In this presentation a scenario is considered in which the robot arm on the space shuttle is used to retrieve a satellite. Due to potential complications on board the shuttle it is necessary to move to a higher orbit while the satellite is being transported to the cargo bay. This condition causes a torque on the robot arm. Because the pilot of the shuttle doesn't want to risk hitting the satellite, the goal is to maximize the distance at which the satellite may be safely retrieved subject to a torque constraint on the primary joint of the arm. The optimal return path that minimizes the torque is also derived using the gradient vector of the torque function. The mathematical model and associated analytical techniques presented here could also be used to model the motion of a robot arm on the Earth under the influence of gravity.

**Ruthi Hortsch, University of Michigan**

**Highly non-injective polynomials maps on the rational numbers**

Consider a polynomial  $f \in \mathbb{Q}[x]$ . For “most” polynomials of degree more than 2, there will be only one rational preimage of a rational point for all but finitely many points. For how many and which  $f$  does this fail? Said otherwise, when can we find infinitely many distinct rational pairs  $(a, b)$  such that  $f(a) = f(b)$ ? Faltings’ Theorem (previously the Mordell Conjecture) tells us that we can approach this question by studying geometric properties of the curve  $(f(X) - f(Y))/(X - Y)$ . In our research this summer, we used this to produce new examples and prove classification results under mild hypotheses. This talk is based on research done this summer with fellow undergraduate Alex Carney under the supervision of Professor Michael Zieve at the University of Michigan.

**Samantha Irvin, Wartburg College**

**Mesoscale Modeling of Quantum Yield in Nanocrystalline Devices**

A new solar material features silicon nanocrystals (nc-Si) within an amorphous silicon (a-Si) matrix. The efficiency with which photons are absorbed and converted to electric current depends on the size, orientation, volume fraction, and interface properties of nc-Si. This investigation resulted in the development of a device level simulator, which predicts quantum efficiency and voltage-current character for nc-Si/a-Si systems. The performance measures are calculated as functions of input photon energy. A number of different interactions were combined to completely characterize the device: optical scattering and absorption; creation of excitons and charge carriers; electrostatic fields; and drift/diffusion of charge carriers.

**Chelsea Johnson, Brigham Young University**  
**Realizable Graphs of Equivalence Classes of Zero Divisors**

In 2007, Sandra Spiroff and Cameron Wickham introduced an equivalence relation  $\sim$  on the set of zero divisors of a commutative Noetherian ring  $R$  by  $x \sim y$  if  $x$  and  $y$  have the same annihilator ideal. Spiroff and Wickham then defined the graph of equivalence classes of zero divisors,  $\Gamma_E(R)$ , to be the graph whose vertices are the equivalence classes determined by  $\sim$ , with  $[x]$  adjacent to  $[y]$  if  $[x][y] = 0$ . In my talk, I will be discussing which of all possible graphs on four, five, and six vertices are realizable graphs of equivalence classes of zero divisors and demonstrate examples of rings which have these graphs. This research was conducted at the 2010 Brigham Young University REU.

**Kimberly Kesting, Fairfield University**  
**Associated Primes of the Cover Ideal**

This past summer I did research in algebraic graph theory, and found some interesting results. In this talk I will introduce the notion of cover of a graph and the cover ideal. I will present how the associated primes of the cover ideal contain certain sub-graphs, in particular odd cycles and centered odd holes. I will also present an original theorem about the contents of associated prime of the cube of the cover ideal,  $\text{Ass}(J^3)$ .

**Katelynn Kochalski, Canisius College**  
**Equitent Problems**

Equitent problems ask what are the surface area minimizing figures that have fixed boundary (extent) and fixed volume (content). We use metacalibration, a new optimization technique, to prove results about the equitent problem in the plane. Specifically, we investigate connecting the vertices of a regular polygon while also enclosing a given area. This presentation is a result of joint research done with Payton Lindsay (SIU Carbondale) at the 2010 Brigham Young University REU.

**Karin Kralicek, Northern Arizona University**  
see **Meredith Daw**

**Andrea Kriz, University of Michigan**  
**An interesting topology on the complex affine line**

Let  $\mathbb{C}$  be the set of all complex numbers. An affine map  $f : \mathbb{C} \rightarrow \mathbb{C}$  is a map given by the formula  $f(z) = az + b$  where  $a, b$  are complex numbers. I proved that there exists a topology on  $\mathbb{C}$  with respect to which continuous maps  $f : \mathbb{C} \rightarrow \mathbb{C}$  are precisely the affine maps. In my talk, I will present the basic steps of the proof, the key point of which is the straightedge-only construction of harmonic conjugates.



**Margaret-Rose Leung, Oregon State University**  
**Samantha Stykel, Luther College**  
**The Effect of Localized Oil Spills on the Atlantic Loggerhead Turtle Population Dynamics**

The loggerhead sea turtle (*Caretta caretta*) is an endangered species with significantly different haplotype frequencies in the regional nesting populations of the Gulf of Mexico and the western North Atlantic Ocean. In this work, we analyze the population dynamics of loggerhead turtles affected by localized oil spill catastrophes. We develop a spatial, stage-classified matrix model and apply it to the three primary nesting regions in the area. Oil spills are simulated deterministically in each nesting region, with oil-induced mortality ranging from 25% to 100% and affecting stage classes either proportionally or equally. We then vary the fecundity and survival parameters uniformly, and use Latin Hypercube Sampling to run stochastic simulations for each nesting region. The results of this study are intended to provide insights into the population dynamics of the Atlantic loggerhead turtles and suggest conservation techniques appropriate in each oil spill case.

**Danika Lindsay, CSU Channel Islands**  
**Disrupting Terrorist Networks**

The organization of terrorist networks suggests that removing key members of a network may be an effective method of disrupting said network. In practice, locating key members is very difficult. We investigate increasing the visibility of a key member of a network by removing a more accessible member, thus increasing communication flow through they key member. Previous research has shown that some nodes are better to remove than others and that there is consistently a best node to remove from a graph. We focus on investigating which nodes, when removed, do not increase communication through the key member. We implement an algorithm to quickly identify those nodes and eliminate them from node removal consideration. This algorithm is compared against a straightforward brute force removal algorithm, and significant improvement in running time is shown for certain graph types. We also create four new types of randomly generated graphs that more accurately represent the characteristics of covert networks.

**Shadiyah Mangru, George Mason University**  
**Investigations in Linear Algebra and Combinatorics related to Biclique Decompositions of Graphs**

We formulate five new propositions related to the Graham-Pollak Theorem. The first four illuminate properties of both biclique edge covers of the edge set of  $K_n$ , and nullspace basis vectors of a matrix representation of such covers. These four propositions motivate the recursively-defined sparse null space basis we present, as proposition five, for a particular subset of matrices of interest in Algebraic Graph Theory.

**Melissa Marchand, California State University, Bakersfield**  
**A Bayesian Time Series Attack on Video Digital Watermarks**

Digital watermarking is a process where information is embedded into digital media. This process is used to help clarify identification of source, creator, owner, distributor, or authorized consumer of a digital media. For the film industry, a digital watermark acts as a source of identification. In the case that a film should leak out illegally, the copyright owner is able to identify the watermark and trace it back to the source of the leak. In this project, we are using both a collusion attack and a Bayesian time series attack to minimize the evidence of digital watermarks without destroying the fidelity of the film.

**Eileen Martin, The University of Texas at Austin**  
**Continuously Moving Parseval Frames on Smooth Manifolds**

Moving bases on manifolds are important in the study of differential geometry and are applied in mathematical physics, but moving bases do not exist on all manifolds, for instance, the sphere. An alternative to a moving basis is a Parseval frame of unit-length vectors. We examine the existence of such frames on the Möbius strip, the Klein bottle, and  $n$ -dimensional spheres. We prove the existence of a continuously moving, unit-length Parseval frame on  $S^n$  when  $n$  is an odd integer. More generally, we investigate the relationship between the existence of a nowhere zero vector field and that of a continuously moving Parseval frame of unit length. One potentially useful tool in studying this relationship is the frame force associated with the frame potential. To better understand this possible method, we are led to a study of the dynamical properties of the frame force.

**Kristen May, Canisius College**  
see **Laura Booton**

**Lauren Mondin, Sam Houston State University**  
**Statistical Analysis of Diagnostic Accuracy With Applications to Cricket**

In the game of Cricket, spectators and officials are interested in making the games as fair as possible. One way to accomplish this is to evaluate the umpires and the correctness of their calls. Estimates of statistical accuracy were used as a basis for comparison. Another way to ensure the objectivity of the game is to be able to, as consistently as possible, determine the winner of the game if it is interrupted for some reason. In the traditional Fifty50 cricket game, the Duckworth-Lewis (DL) method of determining a winner is the preferred procedure. However, with the growing popularity of shorter Twenty20 matches, a new Bhattacharya-Gill-Swartz (BGS) method has also been introduced. We created both frequentist and Bayesian intervals to estimate the true accuracy of each. Using past game data from 2005-2010, we compared the DL and BGS methods using the new accuracy intervals and Receiver Operator Characteristic (ROC) curves, which compare true positives vs. false positives.

**Sarah Mullin, Canisius College**  
**Hannah Yee, Hillsdale College**  
**A Discrete Consideration of Aleksandrov's Projection Theorem**

Aleksandrov's Projection Theorem states that if the areas of projections of two centrally symmetric, convex bodies are equal in every direction, then the two bodies are translates of one another. We will consider the nonexistence of a discrete analogue to Aleksandrov's Theorem given polytopes in  $\mathbb{Z}^2$  where the numbers of projection points of two polytopes are equal in every direction. Counterexamples from previous research on this problem will be discussed, as well as polytopes constructed based on observed characteristics of the known counterexamples. A new proof of an analogue in  $\mathbb{Z}^2$  to Minkowski's Uniqueness and Existence Theorems will also be given, as in continuous space these are key components to the proof of Aleksandrov's Projection Theorem.

**Jocelyn Peck, Brigham Young University**  
**Level 2 Weakly Holomorphic Modular Forms of Negative Weight with Minimal Poles**

In the study of the divisibility of coefficients of the Fourier expansions of modular forms, there is often a need for specific modular forms that mirror the original object of study under Zagier's Duality. These mirroring modular forms can be found by studying the valence formula and applying such restrictions as to make a new modular form out of other well known forms. In the case of level 2 weakly holomorphic modular forms, there is a need for mirroring forms of negative weight with minimal poles at one of the cusps (i.e. zero and infinity) and vanishing at the other. We will discuss the procedure in procuring such forms and produce an algorithm for simplifying the process.

**Catherine Pelland, Pomona College**  
**SET® and disjoint complete caps in  $AG(4, 3)$**

In the card game SET®, it is possible to have 20 cards with no sets. These are the complete caps of the affine geometry  $AG(4, 3)$ . We examine the structure of these caps as well as when it is possible for two caps to be disjoint. We find a decomposition of  $AG(4, 3)$  into disjoint complete caps and determine how such decompositions are permuted by the affine transformations.

**Regina Pollack, Canisius College**  
**Proof of Fermat's Last Theorem when  $n$  is 4**

Pierre de Fermat brought about much consternation over the past couple of centuries because his infamous last theorem was left without a proof. Stemming from several aspects number theory, Fermat's last theorem states that there are no positive integer solutions of the equation  $x^n + y^n = z^n$  for  $n$  greater than two. Countless mathematicians, adamant about proving this theorem, greatly contributed to a final answer, but none were able to fully succeed until Andrew Wiles' proof appeared in the late 1990s. One such important contribution was the proof of this theorem for the case  $n = 4$  which was indirectly described by Fermat himself. This talk will discuss the proof of the case when  $n = 4$  and will illustrate Fermat's strategy of the method of infinite descent, leading to a contradiction.

**Emily Poole, University of Arkansas**  
**Weak Allee Effect, grazing, and bifurcation curves, Part II: Analytical Results**

We study a one-dimensional reaction-diffusion model arising in population dynamics where the growth rate is a weak Allee type. In particular, we consider the effects of grazing on the steady states and establish analytically the existence of an S-shaped bifurcation curve of positive solutions for certain ranges of the grazing parameter. We obtain our results via the quadrature method.

**Christine Rakowski, Daemen College**  
**A Piece of Paper and a Pair of Scissors**

Given an arbitrary region in the plane and a point in the interior of the region, is it possible for a line through this point to split the region in two regions of equal area? Or in two regions with areas  $p\%$  and  $(100 - p)\%$ ? One can explore this question for various well-known geometric shapes, such as the circle and equilateral triangle.

**Stephanie Reed, University of South Dakota**  
**Controlling Plague Among Prairie Dogs**

Prairie dogs, once abundant across the Great Plains, are now faced with Sylvatic Plague known as Bubonic Plague in humans. Though some see prairie dogs as unnecessary nuisances, the once thought extinct Black Footed Ferret has successful reintroduction in South Dakota. The Black Footed Ferret feeds almost exclusively on prairie dogs; however, if the prairie dogs are dying, the Black Footed Ferrets are losing prey and risk the chance of contracting the disease as well. Studies show fleas are the main transmitter of the disease. By scaling logistic differential equations of prairie dogs and fleas, we were able to find four equilibria: (1) Trivial, (2) Only healthy prairie dogs, (3) Only healthy prairie dogs and healthy fleas, and (4) Coexistence of healthy and plagued prairie dogs and fleas. After studying the transfer of disease within each colony, the transfer of disease between two separate colonies is modeled using the basic logistic growth models and adding a travel function based on distance.

**Bonnie Roberson, Mississippi State University**  
**Weak Allee Effect, Grazing, and S-Shaped Bifurcation Curves Part I (Computational Results)**

We study a one-dimensional reaction-diffusion model arising in population dynamics where the growth rate is a weak Allee type. In particular, we consider the effects of grazing on the steady states and discuss the complete evolution of the bifurcation curve of positive solutions as the grazing parameter varies. We obtain our results via the quadrature method and Mathematica computations. Our computational results exhibit the occurrence of an S-shaped bifurcation curve for certain ranges of the grazing parameter. We also provide the complete evolution of the bifurcation curve of positive solutions for logistic as well as strong Allee type growth rates.

**Perla Salazar, Kansas State University**  
**Extinction Times of Solutions to a Degenerate**

We find radial similarity solutions to the equation  $du/dt = \operatorname{div}(\nabla u/|\nabla u|)$ . Using these solutions, a comparison theorem, and radial solutions of other authors, we give close upper and lower estimates of the extinction time of entropy solutions for a large class of initial data.

**Madeline Schrier, University of South Dakota**  
**On the Classification of Structurally Stable Flows**

This work is devoted to the problem of finding vector fields with a prescribed, structurally stable behavior. We show a solution in the one-dimensional case. Then we proceed to present the planar version of this problem. Precisely, we show that one can find a vector field with a given number of limit cycles and we will give a brief description of the classification obtained by Artés, Kooij and Llibre in “Structurally Stable Quadratic Vector Fields” in the case of quadratic polynomials without limit cycles.

**Maggie Schroeder, Northern Arizona University**  
see **Brooke Fox**

**Katie Spence, North Georgia College and State University**  
**Undergraduate Research and Student Interest**

NGCSU will increase spending on its Center for Undergraduate Research and Creative Activities (CURCA) as part of an initiative to focus on undergraduate research and creative activities. A Statistics class performed a service learning project for CURCA. The project concerns views and opinions regarding undergraduate research and creative activities that were solicited in a survey. A targeted sample of approximately 8% of NGCSU juniors and seniors was taken. The class, functioning as 5 consulting teams, each with research responsibilities, worked on developing social science constructs. The class ran multivariate statistical analyses to determine how a large-scale undergraduate research and creative activities initiative would be received by students across campus. The multivariate tests conducted include factor analysis and scale validation procedures, multiple regression, ANOVA's, and non-parametric tests.

**Faye Stevens, Mount Holyoke College**  
**Jamie Woelk, Western State College**  
**Rank Disequilibrium in Multiple-Criteria Evaluation Schemes**

Researchers in the field of conflict resolution have coined the phrase “rank disequilibrium” to refer to situations in which “there are multiple criteria for assessing people’s merit or contributions, and some people are higher on one criterion and lower on another criterion than others.” Rank disequilibrium is known to be a significant cause of organization conflict, in large part due to self-serving biases. In this talk, we will present a mathematical model of rank disequilibrium. Through this model, we will explore desirable properties of evaluation schemes and consider ways in which conflict may be avoided.

**Samantha Stykel, Luther College**  
see **Margaret-Rose Leung**

**Ngoc Thai, Truman State University**  
**Ant Foraging Behavior in the Presence of Climate Change**

Ants are often described as indicator species because they are highly sensitive to variations in the environment, such as climate change. Ants also are a keystone species since sudden drastic changes in their populations could have a devastating impact on other species in the ecosystem. To understand the impact of climate change in a competition between two ant species, we constructed a two-scale model of ant foraging and colony dynamics for hot-tolerant and cold-tolerant ants. The small-scale model used an agent-based approach to simulate two species of ants competing on a daily basis. Functions were then fitted to the average daily food intake for the respective species. The large-scale model was comprised of difference equations for the food controlled by each species and the two ant populations and tracked food and population dynamics over a time scale of years based on foraging behaviors that could be affected by the average daily temperature. Our results showed the possibility of a switch in numerical dominance from cold-tolerant ants, which prefer to forage in colder temperatures, to hot-tolerant ants, which have a higher temperature threshold for foraging. We also modeled the scenario of temperature rising, then falling back to current levels, and observed that natural equilibriums could in some cases be restored, but only if neither of the populations were wiped out during temperature rise.

**Anita Thomas, Illinois Institute of Technology**  
**Computer-Assisted Graph Theory**

A  $k$ -tree is formed from a  $k$ -clique by iteratively adding vertices and incident edges to form new  $k + 1$  cliques. Previously proved by Chappell and Pelsmajer is, for all  $k$ -trees,  $k < 4$ , there exists a vertex subset,  $S$ , that induces a subgraph where all vertices are of degree at most 1 and  $|S| \geq \lceil 2n/(k + 2) \rceil$ , where  $n$  is the order of the  $k$ -tree. Expanding on Chappell and Pelsmajers work, this research considers all  $k$ -trees,  $k > 3$ , with these same conditions. Our approach involves computer-assisted search and classification of  $k$ -trees and specific induced subgraphs that meet the conditions. After reviewing generated graphs, we looked for patterns and attempted to create a proof by cases followed by another proof to show that all  $k$ -trees with  $k > 3$  fit at least one of these cases. Three cases have been identified and proved so far and the rest are a topic for future study.

**Silviya Valeva, Mount Holyoke College**  
**Cops and Robbers on Planar Graphs**

In the game of *Cops and Robbers* on a graph  $G = (V, E)$ ,  $k$  cops try to catch a robber. On the cop's turn, each cop may move to a neighboring vertex or remain in place. On the robber's turn, he moves similarly. The cops win if there is some time at which a cop is at the same vertex as the robber. Otherwise, the robber wins. The minimum number of cops required to catch the robber is called the *cop number* of  $G$ , and is denoted  $c(G)$ . The game of Cops and Robbers has applications in robotics and in search and rescue operations.

A classic result of Aigner and Fromme shows that if  $G$  is planar then  $c(G) \leq 3$ . We characterize the following families of planar graphs as having  $c(G) \leq 2$ : series parallel graphs, outerplanar graphs, maximal 2-outerplanar graphs, and maximal planar graphs with maximum degree at most 5. We also show that every graph  $G$  with  $|V| \leq 9$  has  $c(G) \leq 2$ . This bound is tight, since the Petersen graph (on 10 vertices) requires 3 cops. This is joint work with Aaron Maurer (Carleton College) and John McCauley (Haverford College).

**Fan Wei, Massachusetts Institute of Technology**  
**The Weak Bruhat Order and Separable Permutations**

Let  $S_n$  be the symmetric group of all permutations of  $1, 2, n$ , partially ordered by the weak Bruhat order. Thus for any permutation  $w \in S_n$ , its rank  $\ell(w)$  is the number of inversions in  $w$ . The rank-generating function of  $S_n$  is

$$F(S_n, q) = \sum_{w \in S_n} q^{\ell(w)} = (n)!,$$

where  $(n)! = (1)(2) \cdots (n)$  and  $(i) = 1 + q + \cdots + q^{i-1}$ .

For any  $w$ , we define two graded posets:  $\Lambda_w = \{v : v \leq w\}$  (let the rank of  $v$  be  $\ell(v) - \ell(w)$ ),  $V_w = \{v : v \geq w\}$ . In  $V_w$ . Usually the generating functions of the two intervals are messy. But we will show that if  $w$  is separable, i.e., 3142-avoiding and 2413-avoiding, then

$$F(\Lambda_w, q)F(V_w, q) = (n)!.$$

Moreover, we define a bijection  $\phi : \Lambda_w \times V_w \rightarrow S_n$  satisfying  $\ell(u) + \ell(v) - \ell(w) = \ell(\phi(u, v))$ , and we find an explicit formula for  $F(\Lambda_w, q)$  and  $F(V_w, q)$ . We also show that  $\Lambda_w$  and  $V_w$  are rank-symmetric and unimodal. My advisor for this research is Richard Stanley.

**Kendra Weiss, Metropolitan State College of Denver**  
**A Play on Words**

Coxeter groups can be described using a Coxeter graph, a Coxeter matrix, as transpositions, or words.  $B_3$  is a group generated by 3 transpositions related to a permutation cycle of length 7. This group can also be described as reflections of a cube. In this talk, we will explore the Coxeter group  $B_3$  geometrically, with words, and with transpositions. A Hasse diagram explaining how the words were built up and how they relate to each other will also be presented.

**Jamie Woelk, Western State College**  
see **Faye Stevens**

**Alexandra Wood, DePaul University**  
**Sorting Affine Permutations**

We study reflection length for affine permutations. In particular, we show that elements of  $S_n$  have reflection length at most  $2(n - 1)$ . We also examine what it means to have a particular reflection length, and attempt to describe a property that we call  $k$ -minimality.

**Hannah Yee, Hillsdale College**  
see **Sarah Mullin**

**Amanda Zeringue, Colorado State University**  
**Finding a Near-Optimal Multihomogeneous Structure of a set of Polynomial Equations**

Existing symbolic methods for solving polynomial systems cannot handle the large systems that arise in most real-world applications. The methods of numerical algebraic geometry are far more efficient. The amount of work needed to solve a polynomial system numerically is a root count. One estimate of these root counts is the Bézout bound. A very sophisticated bound is called the mixed volume. The former is simple to compute but often extremely high; the latter is very difficult to compute but often nearly sharp. The multihomogeneous root count is often close to sharp without the high computational cost. We have produced a novel algorithm, relying on graph theory and some simple arguments, that will often produce a near optimal root count. We are able to predict with some degree of accuracy which particular variable partitioning will yield the most efficient root count. Our ultimate goal is to produce reasonable heuristics for finding near-optimal root counts for arbitrarily large polynomial systems.

**Maren Zobott, Northern Arizona University**  
**Using Technology to Transform Established Pedagogy in a Mathematics Content Preparation Course**

This study investigates the application of online tools such as Camtasia, wikis, and shared documents in conjunction with tools such as a smartboard, the TI Navigator networking system, digital pens and other interactive media in a mathematics content preparatory class. Procedural-based instruction to be moved into the online environment, thus freeing classroom time for more student-centered investigations, problem solving, communication, and collaboration. Three sections of a university level “math for elementary teachers course will be involved, to varying degrees, in this quasi-experimental design. Student outcomes will be measured with respect to content knowledge, technological pedagogical understanding, and engagement. Implications of findings will focus on how face-to-face and online technologies can be used to restructure established pedagogy in the mathematics classroom, and how such changes in instruction impact student learning and engagement, classroom pedagogy, and curriculum design.