# The Fourteenth Annual Nebraska Conference for Undergraduate Women in Mathematics

## January 27 - January 29, 2012

## POSTER ABSTRACTS

## Posters by Undergraduate Students

**Cassandra Ahrens, St. Cloud State University**
**Karri Miller, St. Cloud State University**
**Undergraduate Learning Assistants**

An Undergraduate Learning Assistant (ULA) is an undergraduate student serving as a teaching assistant for undergraduate courses. This presentation will focus on the experiences of ULA's for mathematics and statistics courses, highlighting the impact on student learning.

**Allison Beemer, Whitman College**
**Minimizing the Entropy of $\mathbb{Z}^2$ Actions on Toral Automorphisms**

Considering a matrix $A$ in SL(3, $\mathbb{Z}$) that corresponds to an automorphism of the torus $\mathbb{T}^3$ and has distinct eigenvalues and a characteristic polynomial that is irreducible over $\mathbb{Q}$, and a matrix $B$ which is chosen from a carefully constructed subset of the centralizer of $A$, this project seeks to minimize the entropy of the $\mathbb{Z}^2$ action $A^nB^m$ where $n$, $m \in \mathbb{Z}$. Entropy will be measured using the sum of the positive Lyapunov exponents associated with $A^nB^m$. For each $A^nB^m$, we can create a hexagon in the $n-m$ plane using the three Lyapunov exponents of the matrix. The area of this hexagon is inversely proportional to the entropy of the action. By looking at the area of the hexagon, and the case in which we fix $A$ and let $B$ range, we find that there exists a $B$ that minimizes the entropy of the action.

**Kileen Berry, Berry College**
**The Degree/Diameter Problem**

A look at an open problem in graph theory called the Degree/Diameter Problem. The Problem is looking for the largest graph that satisfies given degree and diameter pairs. We will look specifically at the variation of the problem looking at only planar and regular graphs. The goal is to decrease the bounds of the 3-regular planar graphs of diameter 4.

**Anna Broido, Boston College**
**Kathryn Link, Bryn Mawr College**
**A Comparison of Computational Efciencies of Stochastic Algorithms**

In this paper, we investigate three particular algorithms: a stochastic simulation algorithm (SSA), and explicit and implicit tau-leaping algorithms. To compare these methods, we used them to analyze two infection models: a Vancomycin-resistant enterococcus (VRE) infection model at the population level, and a Human Immunodeciency Virus (HIV) within host infection model. While the rst has a low species count and few transitions, the second is more complex with a comparable number of species involved. The relative efciency of each algorithm is determined based on computational time and degree of precision required. The numerical results suggest that all three algorithms have the similar computational efciency for the simpler VRE model, and the SSA is the best choice due to its simplicity and accuracy. In addition, we have found that with the larger and more complex HIV model, implementation and modication of tau-Leaping methods are preferred.

**Shannon Bryce, Kennesaw State University**
**Adaptive Interpolation of Hyperbolic Functions by Linear Splines in L2-error: Local Estimate**

Splines are piecewise polynomial functions. Due to their simplicity, approximation by various types of splines is one of the standard procedures in many applications (computer-aided geometric design, image processing, numerical solutions for partial differential equations etc.). In all these applications, there is a standard distinction between uniform (mesh elements don't vary much) and adaptive (mesh adjusts to the given function) methods of constructing a mesh to build splines. In the uniform case, the domain of interest is decomposed into a partition where elements do not vary much. However, clearly more accurate adaptive methods are highly nonlinear and no polynomial time algorithm exists to provide an optimal approximant for each given function. Therefore, the next natural question would be to construct asymptotically optimal sequences of partitions (that are triangulations when we use linear splines) and interpolating splines on them. To that end we first need to find a triangle that is locally (for some small region) optimal. In this poster we shall discuss how to find the optimal shape of the mesh element triangle in the case of approximating the bivariate functions with negative curvature by interpolating linear splines, and the approximation error on it.

**Teagan Bryce, Kennesaw State University**
**Adaptive Interpolation of Hyperbolic Functions by Linear Splines in L2-error: Global Estimate**

This continues the "Adaptive Interpolation of Hyperbolic Functions by Linear Splines in L2-error: Local Estimate" poster by Shannon Bryce. Once we have found the local L2-error of approximating the bivariate functions with negative curvature by interpolating linear splines, we will put it together to obtain the global estimate for the optimal error. We shall discuss a sketch of an algorithm to construct asymptotically optimal sequences of triangulations and will present the asymptotics of the optimal error.

**Amanda Coons, Clarkson University**
**Communicating with Chaos**

Chaotic systems are deterministic systems that have unpredictable and complex behaviors. They are characterized by high sensitivity to initial conditions and system parameters. It has been found that, under certain conditions, when two identical chaotic systems are coupled together, their outputs will synchronize even though they start with different initial conditions. Research has been conducted to explore the possibility of using the synchronizability of chaotic systems to transmit secure messages. This research uses the Lorenz system to explore possible ways of transmitting a message more accurately through multiple systems.

**Emma Cutler, Bowdoin College**
**Climate Modeling: Interactions between Temperature and Carbon Dioxice**

From ice core data, it is known that over the past 400,000 years, annual average surface temperatures around Antarctica have oscillated between -8oC and 4oC with a period of approximately 100,000 years and that atmospheric CO2 concentrations are correlated with these temperature changes. Although many factors influence climate, simple models can give insight into how these factors interact. The model introduced by Budyko (Tellus, 1969) is a simple climate model, but it does not allow atmospheric CO2 concentrations to change. Over long time scales, the concentration of CO2 in the atmosphere is controlled by constant outgassing of CO2 from volcanic activity and the removal of CO2 by the weathering of silicate rocks at a variable rate. I developed a simple weathering rate law that would provide a reasonable approximation for the weathering rate throughout geologic time in climates different from that of the present Earth, so as to explore the interactions between CO2 and temperature.

**Tetyana Dewland, University of Mississippi**
**Phase Plane Diagrams of Difference Equations**

In this project we analyze the phase plane diagrams of a two dimensional system of first order homogeneous difference equations with constant coefficients. We determine that the geometrical properties of the phase portrait are closely related to the algebraic characteristics of eigenvalues of the coefficient matrix of the system.

**Jennifer Dumdie, University of South Dakota**
**Mathematical Model of Endometriosis**

Endometriosis is an inflammatory gynecological condition found in women of reproductive age, with the prevalence of pelvic endometriosis approaching 6-10% of the female population. It is characterized by the presence of endometrial tissue, the inner lining of the uterus, outside of the uterus, and implantation in the peritoneal cavity can lead to pain, irregular bleeding, and infertility. Although the histologic origin of endometriosis remains obscure, the prevailing hypothesis of retrograde menstruation states that a backflow of menstrual fragments implant, generating lesions. In patients with endometriosis, normal physiological processes such as the inflammatory response, apoptosis, angiogenesis, migration, and invasion are altered in favor of endometrial cell survival. Estrogen, which is found monthly at high concentrations, exacerbates the progression of endometrial lesions. A mathematical model will be developed to understand and analyze the dynamics of some of the quantifiable elements of this disease.

**Lindsay Edwards, Washburn University**
**Planar tilings on which we play the Game of Life**

"The Game of Life" takes place on a grid of cells where any single cell can be "on" or "off" according to certain rules. If, between any two stages, a cell is on and stays on, it is said to have "survived". If a cell is off in one stage and on in the consecutive stage, it is said to have been "born". If a cell was on in one stage but has been turned off in the next stage, it is said to have "died". My exploration changes the shape of the cells from game to game, and the space in which these shapes lay is called the 'background'. The rules that are made for a particular game govern birth, death, and survival, and my challenge is to create extreme rules to see how fast a game will die out or grow over several different backgrounds to compare how the backgrounds affect the death or growth in the game. For my project the possible backgrounds have been narrowed down to the 3 regular and the 6 semi-regular planar tilings, and I have created some over-arching rules for the configuration of the "start stage" of each background, as well as standard rules to guide birth, death and survival.

**Brooke Fox, Northern Arizona University**
**Higher Dimensional Perfect Bricks**

A numerical semigroup is a subset of the positive integers that contains zero, is closed under addition, and has a finite complement. They are useful for exploring certain problems in commutative algebra. A subset of numerical semigroups that have special properties are called perfect bricks. I will be discussing recent developments in the study of perfect bricks with dimension higher than $2 \times 2$.

**Angelica Gonzalez, Whittier College**
**An Infinite Family of Vertex Transitive Directed Strongly Regular Graphs**

Directed strongly regular graphs were first studied by A. Duval in 1988 as generalized objects of strongly regular graphs. Directed strongly regular graphs are obtained from various combinatorial objects, including Cayley graphs, block designs, Hadamard matrices, finite geometries (projective and affine geometries), and partial geometries. There are number of infinite families of directed strongly regular graphs that are constructed on the flags or anti-flags of certain finite incidence structures. These directed strongly regular graphs satisfy certain parameter conditions. We show that the necessary conditions for a finite incidence structure to give rise to a directed strongly regular graph with certain parameter sets are indeed sufficient. We also show that the directed strongly regular graphs that obtained from the flags of certain difference sets and Hadamard designs, have vertex-transitive automorphism groups.

**Alessandra Graf, Northern Arizona University**
**A Comparative Survey of Graceful and Harmonious Labelings**

In 1967, Rosa introduced a type of graph labeling, now known as a graceful labeling, which involves labeling all vertices and edges in a $q$-edge graph such that (a) each vertex is assigned a distinct integer from the set $\{0, ..., q\}$ and (b) edge labels must be distinct and equal to the absolute value of the difference of the two vertices they connect. 13 years later, Graham and Sloane introduced a variation of graceful labeling, known as harmonious labeling, which involves labeling all vertices and edges in a $q$-edge graph such that (a) each vertex is assigned a distinct integer from the set $\{0, ..., q-1\}$ and (b) edge labels must be distinct and equal to the congruence class mod $q$ of the sum of the two vertices they connect. This presentation will compare these two forms of graph labelings as well as provide examples of common graphs that have graceful labelings and harmonious labelings. No prior knowledge of graph theory is needed.

**Lynette Guzman, University of Arizona**
**Optimizing Algorithm for Reliability Assessment of Radial Lifeline Systems**

A prominent reason for finding efficient methods to quantify reliability of radial lifeline systems may be attributed to the susceptibility of the system to large scale failure when a single line segment in the system fails. Proposed methods include Monte Carlo simulation techniques and probabilistic recursive algorithms, which are traditionally limited in their computational efficiency, accuracy, and full analysis of the general case radial lifeline system. This study proposes an algorithm for calculating the complete probability distribution of customer service availability (CSA) for the general case for radial lifeline systems, and explores the sensitivity of components to large scale failure.

**Katie Heaps, Duquesne University**
**Variational Image Denoising and Decomposition Using Duality**

Minimizing the total variation of an image coupled with an appropriate fidelity term has been used in the image processing community for decades as a way to denoise images while preserving important information such as edges and smooth regions. Unfortunately, it also results in staircasing, replacing smooth transitions in color with disconnected flat regions. In this talk, we examine several methods based on the total variation functional for reducing staircasing, including one that uses edge detection to control smoothing and a scheme that minimizes second differences (instead of first) to better preserve ramps. These functionals can be minimized using the dual problem, which allows for a more accurate solution. We will also look at how these methods can be extended to decompose images into their cartoon and texture components. We will observe the effects of these algorithms on a variety of images.

**Shelby Heinecke, MIT**
**Sarah Rasco, SUNY Potsdam**
**Hypercyclic Operators in Hilbert Space**

Consider the space of convergent infinite-dimensional sequences. This is the Hilbert space known as $l^2$. We call a bounded linear operator whose orbit is dense in $l^2$ a hypercyclic operator. Operators whose orbits are dense in a subspace of $l^2$ are known as subspace-hypercyclic operators. In our research, we explore theorems of hypercyclic operators including Rolewicz's theorem, Salas's theorem, Can Le's criterion, among many other fundamental theorems. In our poster, we present our own conjectures and insights to expand on these theorems. We also present a subspace analogue to Salass theorem for hypercyclic operators.

**Kayla Henneman, University of South Dakota**
**Spread of Influenza and Secondary Infection through a Population**

Influenza is the infection associated with the highest mortality in the developed world. This is due to the ever-changing nature of influenza and its annual epidemics. In addition to localized epidemics, influenza has a history of causing worldwide pandemics such as the Spanish flu of 1918, the Asian flu of 1957, and recently the H1N1 pandemic of 2009. During these pandemics, death rates soar. However, healthcare providers, medical experts, and published data from previous pandemics suggest that most of these deaths are caused by a secondary bacterial infection. Many studies have focused solely on the spread of influenza and the result has been an increase in prevention and control of the disease. By studying the spread of influenza and a secondary infection through a system of differential equations, we will gain a better understanding of the disease dynamics which will then lead to increased protection from these diseases.

**Jennifer Herdan, Winona State University**
**Pattern Avoidance and Sorting Algorithms**

This poster consists of recent work done in the area of permutation patterns. One direction is in the area of pattern avoidance over set partitions, particularly 123-avoiding set partitions. We will discuss a method for counting these partitions, particularly for the 3-partition problem. Another area of research is comprised of sorting random permutations with different sorting mechanisms. One of the sorting mechanisms studied was homing, which involves selecting an element and sorting it to its "home". Our results include homing in $n$-dimensions, and homing on multiset permutations, specifically 2-1 permutations.

**Sasha Indarte, Macalester College**
**Estimating Survival Functions for Symmetric Distributions Under Peakedness Order Constraints**

The problem of estimating distribution functions $F$ and $G$ when $F$ is more peaked than $G$ and $F$ and $G$ are symmetric about 0 is addressed by Rojo and Batun-Cutz (2009). These estimators correct for violations in the peakedness order of the empirical distribution functions, $F_n$ and $G_m$. When applying this estimation process to survival functions, the partial censoring of $F$ and $G$ creates greater inaccuracy in these estimators. By using the Kaplan-Meier estimator in place of the empirical distribution functions in the estimators proposed by Rojo and Batun-Cutz (2009), the impact of changing the weights in their benchmark function can then be investigated. Here, two new benchmark functions are proposed for use in their estimators. Through Monte Carlo simulations, the estimators built from the new benchmark functions are compared in terms of mean squared error and bias with the original estimators proposed by Rojo and Batun-Cutz (2009).

**Lara Ismert, Pittsburg State University**
**Rusty Compass Constructions**

Since antiquity mathematicians have developed methods of constructing numbers with only a compass and straightedge. After establishing what numbers are constructible and which are not constructible, a question of further restriction presents itself: What if the compass is rusted to one constant opening? The methods used to construct numbers with a regular compass must be adapted to the new and more difficult restriction of rusty compass constructions. We will show that numbers constructible with a regular compass are also constructible with a rusty compass, but the latter constructions often require more roundabout techniques.

**Esther Jackson, George Mason University**
**Modern Portfolio Theory: Assumptions, Accuracy, and Analysis**

Modern Portfolio Theory attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return. Though commonly known throughout the financial industry, the theory is ill-suited to real market behavior. We examine the underlying assumptions and introduce alternative measures, including nonlinear approaches from manifold learning theory.

**Kiley Johnson, Seattle Pacific University**
**Molly Stubblefield, Western Oregon University**
**Signature Recognition Using Wavelets and Other Methods**

Is it possible to create a machine that will determine if a signature is genuine or forged? The goal of this project is to answer this question in a more modest form, i.e. given a set of a person's signatures, of which only one is genuine, can we develop an algorithm to pick out the genuine one? In order to solve this problem, we must try to quantify the differences between the genuine and forged signatures using mathematics. We employ the use of Haar wavelet decomposition, discrete Fourier transform, and several other methods.

**Sabine Kassis, Mount Mercy University**
**Brittany Shanklin , Mount Mercy University**
**Modeling Print and Electronic Textbook Sales**

Change is in the air. This can be seen clearly in the medium that textbooks are being sold. Our goal was to create a projection that would realistically model print and electronic textbook sales for the next decade. After being presented with this problem at the 2011 Iowa Math Modeling Competition (IMMC), we have built on our solution from the math contest, and improved and expanded the model.

**Ellen Kim, Seattle Pacific University**
**Comparison of Graph Visualization Tools**

Node link diagrams are used very often to represent connections and relationships between entities for analysis. Graphs are commonly used for social network analysis, pattern recognition, data mining, optimization, and can be applicable to many different areas of study. In the development of graph visualization tools certain features have proven to be universally important and vital for analysis. Some features include importing data, layouts/visual components, metrics, and publishing. A high focus is placed on graph layouts addressing the question of how to handle large graphs. Proposed solutions such as simulated annealing processes as well as unique algorithms within existing programs are explored. Through evaluation and comparison of tools a more clear direction for the development of future graph visualization tools can be proposed.

**Rebecca Lelko, Texas Tech University**
**A new algorithm for surface construction**

In this project, we obtain new families of solutions to the Lax-Bonnet system of partial differential equations. We use techniques from the theory of ordinary differential equations and the theory of classical mechanics. Using these solutions, one can obtain new families of surfaces with orthogonal parametrization as solutions to the Gauss-Codazzi equations. An interesting side result is that the Lax-Bonnet equation is a generalization of the celebrated Euler's equations of rigid body mechanics.

**Xiaowei Li, Saint Mary's College of California**
**Rachel Phillips, Saint Mary's College of California**
**Minimum Vector Rank and Complement Critical Graphs**

The minimum rank problem is to nd the smallest rank of a collection of matrices which are related to a given graph $G$. We discuss this problem in the context of minimum vector rank for different classes of simple graphs. In particular, the focus is on certain complement critical graphs and the shared structural features of these graphs. The structural similarities allow for determination of the complement criticality and minimum vector rank of a given graph.

**Kathryn Link, Bryn Mawr College**
see **Anna Broido**

**Joylyn Loveridge, Utah Valley University**
**Mary Petersen, Utah Valley University**
**Victoria Trevino, Utah Valley University**
**Math Projects that Spark the Interest of High School Girls**

We have been participating in Math Girls Rock!, a year-long math engagement program aimed toward female undergraduate UVU math/math education majors, and junior and senior high school girls from the surrounding area. As project assistants of this project, we researched topics in graph theory, geometric probability, and fractal theory. These topics are not generally taught in high school or in our undergraduate courses thus far. Through our research we helped develop meaningful, fun, and engaging math activities and projects. Then we worked with the high school girls on these projects that helped them learn some of the interesting mathematical concepts. In this presentation we will discuss the three projects, talk about our research findings. In addition, we will explain how we mentored the high school girls through the projects: three-edge colored Buckyballs (dodecahedrons), Buffon's needle problem (geometric probability), and fractal curves (with paper folding).

**Sara Melvin, The University of Texas at Tyler**
**Stick and Edge Numbers of Composite Lattice Knots**

In this presentation the authors explore properties of the stick and edge numbers of knots in the simple hexagonal, face centered cubic, and two variations of the body centered cubic lattices. In each lattice the stick and edge numbers are shown to be subadditive with respect to composition. Additionally, upper bounds for the composition of $n$ trefoils are given for each lattice.

**Karri Miller, St. Cloud State University**
see **Cassandra Ahrens**

**Rebecca Miller, University of Central Oklahoma**
**To Be Woman or Not To Be: "The Struggles of Women Mathematicians and How It Has Impacted Mathematics"**

Women. Our society/civilization would not exist without them. So why does society make the doors of math so heavy for women to open, let alone cross the threshold? What struggles have different women gone through to enter these great doors? What impact have their struggles had on the mathematical community? Often we do not realize others' adversities until we run into the conflict ourselves or are made aware of the conflict. In this poster, we have research that will not only bring awareness and knowledge of these struggles and their causes to current and future mathematicians, but also show the importance of closing this gender gap by uncovering the impact this gap has had on the mathematical community. In the words of Sofia Kovalevsky, the greatest female mathematician of the 19th century, "... is it really possible not to stretch out one's hand, is it possible to refuse to help someone who is seeking knowledge and cannot help herself reach its source?... I consider it is my duty to destroy whatever obstacles I can in the paths of others."

**Kelly Montgomery, Michigan State University**
**Cantor Set vs. Tent Map**

The research involves relating two ideas in mathematics: The Cantor Set, which was coined by Georg Ludwig Phillip Cantor in 1883 to the dynamics of the Tent Map. Iterations of the tent map will produce a set that correspond to the removed intervals of the Cantor Set. In this work, we consider the connection between generalized tent maps and the corresponding Cantor Sets that arise through functional iterations. We will also examine the fractal dimensions of the fractal structures and compare them to the fractal dimension of the original Cantor set. Similarities and differences between these sets as well as applications will be explored.

**Mary Petersen, Utah Valley University**
see **Joylyn Loveridge**

**Rachel Phillips, Saint Mary's College of California**
see **Xiaowei Li**

**Elise Prete, Furman University**
**Clarisse Quandt, Furman University**
**Disconnected Gamma Graphs**

As introduced in the paper by Fricke, et al., given a graph $G = (V, E)$, the $\gamma$-graph $G(\gamma) = V(\gamma), E(\gamma)$ is the graph whose vertex set corresponds in a one to one relationship with the $\gamma$-sets, or minimum cardinality dominating sets, of $G$. Two $\gamma$-sets, say $S_1$ and $S_2$, are adjacent in the gamma graph if there exists a vertex $v \in S_1$ and a vertex $w \in S_2$ such that $v$ is adjacent to $w$ in $G$ and $S_1 = S_2 - \{w\} \cup \{v\}$, or equivalently, $S_2 = S_1 - \{v\} \cup \{w\}$. In this poster presentation, the connectivity of $G(\gamma)$ will be explored. It will be shown that if $G$ contains certain structures, $G(\gamma)$ will be disconnected. The conjecture that all disconnected $\gamma$ graphs might contain a certain set of structures will be considered.

**Clarisse Quandt, Furman University**
see **Elise Prete**

**Sarah Rasco, SUNY Potsdam**
see **Shelby Heinecke**

**Ariel Setniker, Western Oregon University**
**Inverse Modeling of Dynamical Systems**

Conditioning likelihoods are typically much simpler to model than the full joint distribution which may be difficult or impossible to find analytically. Conditioning has the potential to improve the identifiability of the estimation problem. We will thus explore modeling with ordinary differential equations and quantify various features thereof, such as frequency and peak amplitude. We demonstrate comparisons of bias accuracy among methods and provide case studies of stochastic switching, bistable oscillation (Terman and Wang, 1995), and predator-prey relationships.

**Brittany Shanklin, Mount Mercy University**
see **Sabine Kassis**

**Molly Stubblefield, Western Oregon University**
see **Kiley Johnson**

**Victoria Trevino, Utah Valley University**
see **Joylyn Loveridge**

**Guanyu Wang, The University of Iowa**
**Tangle Tabulation**

A knot is the image of a circle (i.e, a closed arc) embedded in 3-dimensional space. Tangles are similar to knots, but consist of strings whose endpoints are “nailed down” on the boundary of a 3-dimensional ball. In knot tabulation, knots are tabulated using crossing number (the minimal number of crossings needed to draw the diagram of a knot/tangle). In a similar manner to knot tabulation, we are creating a table of two string tangles ordered/categorized by crossing number. A sequence of numbers is used to represent a tangle which can be visualized by using the software KnotPlot. From there, code is being implemented to generate various invariants, each of which is a quantity that is the same when computed from dierent descriptions of a knot/tangle. A webpage is being developed in which users can create a table of tangles and their different invariants.