

**The Fifteenth Annual
Nebraska Conference
for Undergraduate Women
in Mathematics**

January 25 - January 27, 2013

TALK ABSTRACTS

PLENARY TALKS

Dr. Cathy O’Neil
Independent Consultant and Math Blogger
Mathbabe
What do mathematicians do outside academia?

I will talk about the variety of career options available with a degree in math outside of being a teacher or professor. I’ll expand on the different environments, kinds of feedback, and types of impact you might have on the larger world depending on what you do.

Dr. Rekha R. Thomas
Professor of Mathematics
University of Washington, Seattle
Polynomial and Semidefinite Optimization

Many problems in science and engineering can naturally be modeled using polynomials. In the past decade, there has been a whole new approach to solving polynomial optimization problems using the modern tool of semidefinite programming, a branch of optimization with efficient algorithms that generalizes linear programming. This tool has many applications in areas such as computer vision, robotics, coding theory, computational biology, crystallography and so on. In this talk I will explain the basics of these powerful techniques, several applications, and connections to classical mathematics dating back to David Hilbert in the late nineteenth century.

Talks by Undergraduate Students

Lauren Akin, Freed-Hardeman University
Whitney Turner, Freed-Hardeman University
Leah Wright, Freed-Hardeman University
Data Mining General Education Course Assessments

Our research analyzes general education course data at Freed-Hardeman University, looking for correlations amongst classroom factors. We specifically searched for correlations between performance on several student learning outcomes and varying course components, such as: course, class size, semester, time of day, duration, and outcome in the class. The students general education quantitative assessments were evaluated using Orange data mining software. Ideally, our purpose is to discover and report information to our school that may improve student learning.

Elizabeth Annoni, University of St. Thomas
Nicole Lopez, University of St. Thomas
Classifying knots in open chains

Classical knot theory centers solely around knots formed in closed loops. Therefore in order to study open knots the endpoints must be connected to form a closed loop. The purpose of this project is to analyze and refine methods for classifying knots in open chains. This is achieved by taking points on the sphere and connecting both endpoints to each point to form a closure. The options for choosing points on the surface of the sphere are randomly distributed points, vertices of platonic solids or approximately uniformly distributed points. Due to the inconsistent nature of random distributions, data may be biased and unreliable. Platonic solids have vertices that are perfectly distributed about a sphere; however there are too few solids to be of use. Hence this project analyzes methods for approximating uniformly distributed points on a sphere. Voronoi diagrams were used to compare the methods of distribution and determine the respective areas for each closure point.

Hanna Astephan, University of Michigan
Common Values of Polynomials Over Finite Fields

Let K be the finite field of q elements, K_i its degree- i extension, and f and g polynomials in $K[x]$ of degree at most n . We provide several results and examples about the possibilities for N , where N is the cardinality of the intersection of the image sets $f(K)$ and $g(K)$. For instance, there are positive constants a_n and b_n , which depend only on n , such that either $N < 2n$ or $N > a_n * q - b_n * \sqrt{q}$. Moreover, if $f(K) = g(K)$ and q is larger than some explicit function of n , then there are infinitely many i for which $f(K_i) = g(K_i)$. On the other hand, there are rational functions f, g in $K(x)$ such that $f(K_i)$ equals $g(K_i)$ for even i , but $f(K_i)$ and $g(K_i)$ are disjoint for odd i . Our results depend on various ingredients, including deep group-theoretic results and a new function field analogue of the Frobenius Density Theorem.

Riley Banes, Mount Mercy University
Clara Camarillo, Mount Mercy University
Lauren Hoth, Mount Mercy University
ExploreU@MMU

ExploreU@MMU is a two-week STEM (Math and CS) related program at the Mount Mercy University for middle school girls. Four female undergraduate math and CS majors served as mentors in this program. New math and CS software was used and various math and CS topics covered. In this talk, we will talk about the program and our mentoring experiences.

Shelby Becker, Saint Mary-of-the-Woods College
Using Eye-Gazing Data to Predict Radiologists' Cognitive Behavior during Breast Cancer Screening

Breast cancer is a leading cause of death in women in the US, but early detection greatly improves survival. Research has shown that 10% to 30% of breast cancers are missed and most lesions are visible retrospectively in mammography. CAD systems are used to reduce human error, but it ignores each radiologists perceptual and cognitive needs. These are the two types of diagnostic errors associated with detection of breast cancers. Artificial intelligence techniques are applied to capture the radiologists perceptual and cognitive patterns from the data. The data mining algorithms implemented in Excel, WEKA, and JMP are investigated to determine whether visual search characteristics could be used to predict radiologists decisions. Results show that the algorithms are capable of predicting radiologists cognitive behavior at the group as well as individual level. This research could lead to the next generation of CAD systems that are user-adaptive to each radiologist and case under review.

Amy Been, University of Nebraska-Lincoln
Generating Pythagorean Triples in the Quaternions

Solutions to $x^2 + y^2 = z^2$ are widely explored in the integers; however, much is left to be discovered in other rings, such as the real Quaternions. Our research focuses on the Quaternions and examines conditions under which we can find families of Pythagorean triples in that ring. Our method consists of taking a Pythagorean triple $(a, b, c) \in \mathbb{H}_{\mathbb{Z}}$ and constructing a $t \in \mathbb{H}_{\mathbb{Z}}$ dependent on a, b , and c such that $(a - t, b - t, c + t)$ is a Quaternion Pythagorean triple. To find necessary and sufficient conditions on t , we utilize the results that for $b \in \text{Im}\mathbb{H}_{\mathbb{Z}}$, b^2 is an integer and $b^2 = -N(b)$, where $\text{Im}\mathbb{H}_{\mathbb{Z}}$ denotes a pure imaginary Quaternion.

Beth Bjorkman, Grand Valley State University
The Fixed Points of the Columnar Transposition Cipher

A columnar transposition cipher is an encryption technique that permutes the characters of a message using positions in a rectangular enciphering grid. In this project, we investigate the existence and location of the fixed points of the underlying permutation as they relate to the number of columns and message length used.

Amanda Bright, Westminster College
Minimum Exponential Dominating Sets of Connecting Cycles

The purpose of this research is to explore the behavior of minimum exponential dominating sets of separate and equal cycles. It is well known that the size of a minimum exponential dominating set for a cycle of size $n \geq 5$ is $\gamma_e(C_n) = \lceil n/4 \rceil$. When two cycles of size n are connected at one vertex, we are going to prove the resulting minimum exponential dominating set, $\gamma_e(Q_n) = 2\lceil n/4 \rceil - 1$ for $n \geq 5$. We are also going to discuss the minimum exponential dominating set when two cycles of size $n \geq 5$ are connected by two vertices d apart to be

$$(Q_{nn}^d) = \begin{cases} 2\lceil n/4 \rceil - 1, & \text{when } (d+3) \bmod 4 < (n+3) \bmod 4 \\ 2\lceil n/4 \rceil - 2, & \text{when } (d+3) \bmod 4 \geq (n+3) \bmod 4. \end{cases}$$

We will further look into connecting additional cycles to make a conjecture to generalize the behavior of the minimum exponential dominating set.

Cara Cannon, Hope College
Mathematical Modeling the Growth and Spread of Marram Grass

Populations in Vegetated Sand Dunes The presence or absence of plants critically affects the physical processes that shape and form vegetated sand dunes. This is due to the fact that roots impede erosion, while foliage promotes sand accumulation by slowing down the wind. *Ammophila breviligulata* (marram grass) thrives under conditions of moderate sand burial. This introduces interesting feedback dynamics, because the grass presence impacts accumulation of sand, which, in turn, stimulates plant growth and spread. In this talk we will examine one side of this interaction by mathematically modeling marram grass population dynamics in the presence of sand burial. The model incorporates both local growth and spread through the use of integrodifference equations. Through model simulations we will address important questions relating to vegetated dune dynamics including whether population dynamics alone can lead to the formation of blowout sand dunes.

Stephanie Carter, Pacific University
Tara Walker, Seattle Pacific University
Sarah White, Wartburg College
Word Recognition using Wavelet and Fourier Analysis

Using wavelet and Fourier analysis, we created a voice dependent word recognition program to try and identify three specific words within our own pre-recorded library of recordings. We then attempted to identify the words recorded by other group members using the same algorithm. After this, we adapted our libraries to the analysis of the numerals 0 to 9, with the aim of creating a voice dependent algorithm that correctly identified recorded numbers.

Megan Chambers, Youngstown State University
The Bigger Half: Examining Fair Division

The Fair Division Dilemma, also known as the Cake-Cutting Problem, is a method of resource allocation used to ensure that each party sharing the resource believes that they have received at least a fair share. It is a problem that has been studied extensively by mathematicians for years and has been the topic of many mathematical papers and books. In my presentation, I examine this problem and its many variations, as well as applications of the problem under different conditions. The potential uses for the problem are abundant, and the mathematics behind it are beautiful, not to mention delicious!

Justina Cline, Coe College
Modelling Cancer Stem Cell and Non-Stem Cancer Cell Population Growth

Each year cancer treatment in the US costs more than \$120 billion, with over a million new people diagnosed with cancer. In order to improve cancer treatment techniques, researchers are studying tumor growth and behavior. Based on the Non-Stem Cancer Cell Hypothesis, which postulates the existence of the non-stem cancer cell, the population growth of a heterogeneous tumor can be modeled using a multi-compartment and a 2-compartment differential equation model. We can then analyze the population ratio and the age structure distribution with respect to the parameters. To more realistically model tumor growth, a 1-dimensional partial differential equation model was developed, as a first step towards a multi-dimensional model.

Kathleen Daly, Lewis & Clark College
T-segment Intersection Graphs

The information contained in a graph can be represented in different ways by assigning to each vertex of the graph a set such that the intersection of two sets will be nonempty if and only if their corresponding vertices are adjacent. In this talk we will explore the class of graphs that can be described by representing the vertices as straight line segments and adjacency by segment intersections that occur only at the endpoint of exactly one segment. Both two-dimensional and three-dimensional representations will be considered.

Connie Dennis, Shawnee State University
Cassie Stamper, Shawnee State University
Algebra in Flatland

The classical notions of commutativity in graded algebras correspond to the behavior of the states of physical bosons and fermions when two particles are exchanged. In three or more dimensions these are the only possibilities. However, in two dimensional physics other types of particles are possible states can multiply by some other phase $\zeta^N = 1$, when two particles are exchanged. Such particles are called anyons. The goal of the project was to investigate commutativity and associativity for graded algebras in the anyonic setting. Anyonic commutativity when ζ is a primitive N -th root of 1 was investigated using elementary number theoretic properties of N . Anyonic associativity coincides with the classical notion unless the operation has a non-zero degree. The theory of rewrite systems was used to construct anyonic associative algebras with multiplication of degree $m \neq 0$ freely generated by elements of specified degrees.

Jessica DeSilva, California State University, Stanislaus
Interval-Vector Polytopes

An interval vector is a $(0,1)$ -vector where all the ones appear consecutively. Fixing a dimension, we take the convex hull of certain subsets of interval vectors to form polytopes with interesting properties. We present a number of classes of interval-vector polytopes and prove in increasing generality their volumes and f-vectors. Among these classes are n -dimensional polytopes whose volumes are the n th Catalan number and another whose volumes are the even numbers and whose f-vectors mirror the Pascal 3-triangle.

Jennifer Dumdie, University of South Dakota
Mathematical Modeling of Endometriosis

In patients with endometriosis, normal physiological processes such as the inflammatory response, apoptosis, angiogenesis, migration, invasion and cellular adhesion are altered in favor of endometrial cell survival and implantation. A mathematical model is being developed to analyze the dynamics of some of the quantifiable elements of this disease. This hybrid discrete-continuum model focuses on five variables involved in the process of endometriosis lesion formation. These variables are endometrial cell density (n), chemotactic signal by cytokines (c), adhesion (a), extracellular matrix (f), and matrix-degrading enzymes (m). This model will be used to analyze the effects of specific genes, for example claudin-11, known to be differentially expressed in endometriosis. In conjunction with a gene transfection study, this mathematical representation will provide predictive insight into, along with a way to experimentally verify, which genes may be responsible for the development and progression of this disease.

Alyce Eaton, Grinnell College
NCAA Division III Cross Country Championship Meet Selection

Each year, the top DIII college cross country teams in the nation are invited to compete in the DIII NCAA Championship meet. A Coaches Committee chooses, under certain parameters, which teams to send on to this national meet from each region. But how can coaches choose which team is "best"? Committee members often have to use incomplete information to compare teams that have not competed against each other, while taking into account individual runners, time of season, weather, course make-up, and more. Several methods have been presented to quantitatively rank teams to make this process easier. We looked at Regional and National meet data from the past 5 years to determine how well these methods work and eventually to propose our own method.

Missy Gaddy, Wofford College
Restoration and Analysis of Apollo Lunar Data

The Lunar Ejecta and Meteorites (LEAM) Experiment on Apollo 17 was designed to measure hypervelocity particles that collide with the surface of the Moon. Original data analyses from the LEAM experiment yielded unusually high numbers of low-velocity impacts during the passage of the terminator. LEAM data was used as a key piece of evidence for dust levitation theories at the terminator, a now highly studied phenomenon. Recently, a question about electrical noise on the LEAM instrument has arisen, and there is some speculation that noise generated by the heaters and other instrumentation turning on and off may have influenced the results. The current analysis examines the LEAM data more closely to see if there are correlations between the unexpected number of events and the times that the power levels were fluctuating. Analyses have revealed unusual patterns in the accumulators of the UP, EAST, and WEST sensor. Additional channels of Housekeeping data can be studied to confirm the presence of interference.

Selina Gilbertson, Northern Arizona University
Realizability of a Critical Portrait

Given a rational function f with fixed critical points, the associated branch data of f refers to a set of partitions of the degree of f , with each partition determined by the local degrees of points in the preimages of a corresponding distinct critical value of f . The critical portrait of f is determined by a partition of the multiplicities of the critical points of f . It is known that a critical portrait is realizable if it comes from a connected planar multigraph G , where each critical point corresponds to a vertex of G and each vertex has degree equal to the multiplicity of the associated critical point. It is also known that necessary conditions for the realizability of a critical portrait are that the associated branch data is realizable and the number of distinct critical points is at most the degree of the function. This talk will discuss a graph-theoretic proof to conclude that these conditions are also sufficient for the realizability of a critical portrait.

Erica Gilliland, Butler University
Enumerating Unlabeled Trees

One of the earliest papers on graph theory was written by Euler in 1736. Since then, graphs have been studied by the likes of Foster, Coxeter, Biggs, and Peterson. Applications extend beyond the world of pure mathematics into the areas of engineering, computer programming, chemistry, and more. Yet, after three hundred years of intense study, there are many problems that are open or in need of improvements. In this talk, we will examine an improved algorithm that we developed for counting the number of unlabeled trees where the degree of each vertex is less than or equal to four, which translates into counting the number of isomers of alkanes.

Alessandra Graf, Northern Arizona University
A New Graceful Labeling for Pendant Graphs

A graceful labeling of a graph G with q edges is an injective assignment of labels from $\{0, 1, \dots, q\}$ to the vertices of G such that when each edge is assigned the absolute value of the difference of the vertex labels it connects, the resulting edge labels are distinct. A labeling of the first kind for coronas $C_n \odot K_1$ occurs when vertex labels 0 and $q = 2n$ are assigned to adjacent vertices of the n -gon. A labeling of the second kind occurs when $q = 2n$ is assigned to a pendant vertex. Previous research has shown that all coronas $C_n \odot K_1$ have a graceful labeling of the second kind. In this presentation we will show that all coronas $C_n \odot K_1$ with $n \equiv 3, 4 \pmod{8}$ also have a graceful labeling of the first kind. No knowledge of graph theory required for this talk.

Lindsay Grayson, Northern Kentucky University
Assessing the Chaotic Nature of Interstellar Magnetic Fields

High energy cosmic rays serve an important role in the formation of stars and planets. As charged particles, the rays are accelerated when traveling through a magnetic field and the turbulent nature of the field causes the propagation of these rays to be chaotic. As such, two identical particles injected into the field at slightly different starting positions will follow divergent paths. This divergence can be quantified by Lyapunov exponents. Models for the MF consist of sums of N randomly directed Alfvén waves, as predicted by linear magneto-hydrodynamic theory. We calculated the Lyapunov exponents numerically for the MFs and found one positive exponent each time, indicating the chaotic nature of the field lines. We ran simulations to obtain a distribution of maximal Lyapunov exponents (MLEs) for each N . We found that the mean MLE increases with N and plateaus at approximately $N = 100$. Thus a model of 100 waves adequately describes the chaotic nature of the MF.

Natalie Guerra, Hood College
Megan Rodriguez, Hood College
Conceptual Understanding of Integer Operations

In this research project, we investigated 7th-graders' misconceptions about working with integers, while deepening our own mathematical and pedagogical understanding of them. Together with two faculty members at Hood College, we researched the literature on student understandings of integer computation, developed interview protocols, and then conducted 20 interviews of 7th grade students using a Smartpen, a ballpoint pen with an embedded computer and digital audio recorder. We transcribed and analyzed the interview data and used that data to identify student needs. Then we produced "pencasts" – interactive documents containing audio and graphics – for middle school teachers, helping them to focus on student needs and teach integer computations conceptually, without relying on gimmicks. We will describe our research process and our findings.

Cassy Hanson, Western Oregon University
Modeling Newcastle Disease in Wild Parrots

We begin with a commonly known SIR disease model, and demonstrate the process of adapting it to a specific wild life population. Our model considers the "what if" scenario of a particularly virulent disease, specifically Newcastle Disease, finding its way into a species of wild parrots in the jungles of Peru. We find that population density along with two stages of the disease that respond differently in young versus mature birds, make this type of model especially challenging, but we expect it will lend itself nicely to future modeling situations.

Kristen Heaney, La Salle University
Rainbow Coloring

Rainbow connectedness is a concept in graph theory introduced in 2008 by Chartrand et al. The rainbow connection number of a connected graph G is the minimum number of colors required to color the edges of G in such a way that every pair of vertices is connected by a rainbow path, a path with no two edges assigned the same color. The strong rainbow connection number requires a rainbow shortest path between every pair of vertices. The motivation for this topic is the issue of secure transfer of information. A thorough investigation of rainbow and strong rainbow connection numbers within the families of cycles, wheels, and other similar graphs was conducted. These graphs include helm, web, and various ladder graphs. This work reveals conclusions drawn not only about the graphs respective connection numbers, but also about their optimal coloring strategies. In addition, building on the rainbow connection number of a graph, a new concept is presented that introduces an additional level of optimization.

Nikki Holtzer, Stetson University

A Solution to the Model of an Oboe Reed Under the Presence of a Weak Quadratic Nonlinearity

Relatively simple models are known to be sufficiently valid for all members of the reed family of instruments. However, the conception of this model is based on the assumption that the bore of these instruments and the reed can be described independently. The same cannot be said for the model of a double reed instrument. The addition of a nonlinear restoration force is necessary to consider when analyzing this model. Utilizing perturbation techniques, we compute an analytic solution for the model of an oboe reed under the presence of a weak quadratic nonlinearity.

Lauren Hoth, Mount Mercy University

see **Riley Banes**

Kelsey Houston-Edwards, Reed College

Minimum Modulus Problem

Our research is inspired by Erdos' minimum modulus problem: Given any natural number c , can one construct a covering system using only moduli greater than c ? We explore various ways to minimize such coverings: What is the minimum number of moduli necessary to construct a covering? What is the minimum possible greatest modulus necessary to create a covering?

Lara Ismert, Pittsburg State University

An Inequality for an Isosceles Triangle

In this talk we will discuss a partial result to Problem #11646 from the MAA Monthly. Using coordinate geometry, we will outline a proof for an inequality for any isosceles triangle involving the inradius, circumradius, altitudes, and radii of three circumcircles constructed with the vertices and angle bisectors of the triangle. Our first step will be to solve for each of these parts of an isosceles triangle using only two variables - the base and the height - and we will then show the inequality is true using basic principles from Calculus.

Kara Karpman, Duke University

Modeling Mucociliary Clearance

Mucociliary clearance is a process whereby cilia lining the airways propel mucus and entrapped particles away from the lungs. Understanding this process is critical since clearance protects the lungs from bacteria, viruses, and other harmful substances. We formulated a fluid model to simulate mucociliary transport of a single particle. The method of regularized Stokeslets, in combination with the method of images, was used to compute the velocity field in the mucus layer due to ciliary beating. The advective and diffusive motion of the particle was then modeled using the forward Euler method. Our simulation is used to gain insight into which biological factors impact mucociliary clearance, including ciliary spacing, ciliary phase differences, and particle diffusivity.

Keri Kodama, Seattle University
Stability of Traveling-Wave Solutions to the Whitham Equation

Two equations that model surface waves in shallow water are the Korteweg-de-Vries (KdV) equation and the Whitham equation. KdV was first derived in 1895 and has been studied extensively. It is an effective model for long waves, but it fails to accurately model short waves. Bottman and Deconinck (2009) proved that all periodic traveling-wave solutions of KdV are linearly stable. The Whitham equation is a generalization of KdV that addresses this issue and should accurately model the evolution of waves of all lengths. The Whitham equation was introduced in 1967 and has not been studied in as much detail as KdV. Ehrnstrom and Kalisch (2009) established that Whitham admits periodic traveling-wave solutions of a wider variety than KdV. We examine the stability of this broad class of traveling-wave solutions to the Whitham equation using linear stability analysis to determine whether these solutions are likely to maintain their profile as they evolve in time.

Taryn Laird, Northern Arizona University
Trees of Irreducible Numerical Semigroups

A 2011 paper by Blanco and Rosales describes an algorithm for constructing a directed tree graph of irreducible numerical semigroups with fixed Frobenius numbers. After providing background information, we will explain the algorithm, construct some examples, and state several conjectures about these directed tree graphs.

Shannon Lane, College of St. Benedict
The Use of the Stratification Propensity Score Method in an Observational Study

Propensity scores have been widely used in efforts to balance the distribution of the covariates in observational studies. This paper uses the stratification propensity score method and compares three different size strata: five, ten, and twenty. These methods use data from the 2009 Medical Expenditure Panel Study (MEPS) in attempt to balance the covariates and determine differences among the different size strata.

Nicole Lopez, University of St. Thomas
see **Elizabeth Annoni**

Amanda Luby, College of St. Benedict
Salmonella Surveillance

This project focuses heavily on using Bayesian statistical methodology to determine change-points using a Markov Chain Monte Carlo computational method as well as a Bayesian Poisson Analysis. Furthermore, a simulation study was developed to evaluate how well these methods were able to successfully detect Salmonella outbreaks, as well as how different factors affect these results. Results showed that the Bayesian Poisson analysis was better at detecting outbreak trends of Salmonella, including the individual strain Salmonella Infantis. The simulation study showed that the two most important factors in detecting an outbreak are the probability of staying in an outbreak if one is already occurring (p) and the probability of missing an outbreak that happened (α).

Brianna Lynn, George Mason University
Static Two-Dimensional Sponge Deformation

The shape of elastic physical objects can be modeled by partial differential equations. We describe an approach to determine the elasticity parameter of a sponge under compression from experimental measurements. After solving the partial differential equations, we used a nonlinear optimization technique to determine the elastic parameter.

Kristin Mara, Winona State University
Semiparametric Regression for Manufacturing Parts Data

One of the approaches to model the smooth function in a nonparametric model is to approximate it by adopting adequate basis functions. In this research project, we approximated the smooth function by a truncated polynomial basis with degree 2, which contains the polynomial basis and the splines constructed by knots. After we fix the number of knots, the function is estimated by well-known methods such as ordinary least squares, penalized spline regression and linear mixed model. We propose our version of Bayesian penalized spline, which provides comparable results. The prior distribution is chosen to be objective so it will minimize the influence to the posterior distribution and maintain the advantages Bayesian statistics provided. After we fit a nonparametric model, we will look at a semiparametric model. This semiparametric model will combine our nonparametric model with a categorical variable. We will use the AIC to compare those methods proposed through a simulated data set and a manufacturing data set.

Katie McKeon, University of Nebraska - Lincoln
Voltage Assignments for Graph Products

A voltage assignment function maps edges from a graph to elements in a group. A voltage graph is then the combination of a graph G , a group A , and the voltage function α from $E(G)$ to A . This voltage graph can be used to generate a lift graph, \hat{G} such that \hat{G} encodes information about both the group operation and the original graph G . In this talk, we investigate the relationship between a graph product of two lift graphs \hat{G} and \hat{H} and the lift of a product of two graphs G and H . Specifically, if $*$ denotes some graph product operation (e.g. the Cartesian product, the direct product, etc.), we attempt to derive a voltage graph for $G * H$, so that $\widehat{G * H}$ is isomorphic to $\hat{G} * \hat{H}$.

Claire Merriman, Vanderbilt University
Properties of Pythagorean triples over the integer quaternions and a partial parameterization

In the rational integers, there is a well known parameterization for all primitive Pythagorean triples. This parameterization can only be extended to commutative rings. However, there is a lesser known parameterization where $r^2 = st + ts$, then $(r + s, r + t, r + s + t)$ is a Pythagorean triple in any ring. This talk will explore when $r^2 = st + ts$ where $r, s, t \in \mathbb{H}_{\mathbb{Z}}$, the ring of quaternions with integer coefficients. Additionally, it will look at some properties of Pythagorean triples in this ring that could be helpful to finding a complete parameterization.

Rosanna Mersereau, Western Oregon University
Magic Cayley-Sudoku Tables

A Cayley-Sudoku Table (C-S Table) is the Cayley table of a finite group arranged (unconventionally) so that the body of the Cayley table is divided into blocks containing each group element exactly once, as in a sudoku puzzle. Dr. Michael B. Ward and his students introduced C-S Tables in *Mathematics Magazine* **83** (2010) pp. 130-139. In this talk we introduce Magic C-S Tables in which each block of the C-S Table is a magic square. That is, the (group) sum of the entries in each row, column, and diagonal of each block equals the same fixed group element. Examples and theorems from the first half of the 6-month research period will be presented.

Alice Nadeau, Grinnell College
On the Folding of L-shapes

Recent work in computational geometry has begun to address the question: Can a given convex polyhedron be unfolded into a simple polygon and then refolded into any other convex polyhedron? One facet of this question investigates the space of polyhedra that can be realized from folding a given polygon. While the folding of convex polygons is fairly well understood, there are still many open questions regarding the foldings of non-convex polygons, and current methods for determining their polyhedral realizations are computationally inefficient. We analyze these realizations and their volumes derived for the polygonal family of 'L-shapes,' parallelograms with a parallel copy removed from a corner. This family includes the special case of the rectangle which has been previously studied. We study the family of polyhedra that unfold to a common L-shape and provide developments in the topics of maximal volume polyhedron, edge relocation, diagonal flipping, and topological space.

Cara Nickolaus, Washington University in St. Louis
Intrinsic Linking in Graphs

A graph is intrinsically linked if every embedding of the graph contains some pair of cycles that form a non-split link. Robertson, Seymour, and Thomas originally proved the minor minimal set of intrinsically linked graphs to be the Petersen Family Graphs in 1995. We seek strategies to reprove this result in a simpler way by defining new concepts such as weak flexible planarity and flatness and by exploring the conflict graphs of maximally planar subgraphs, analogous to Tutte's work with conflict graphs associated to cycles. We also discuss results regarding permutation graphs and intrinsic linking of the join of a graph and two disjoint vertices.

Diana Oliff, Rutgers University New Brunswick
Number of Solutions to Quadratic Equations Over 2 by 2 Matrices over the Quaternions

An n th degree polynomial over the real numbers has at most n solutions. However, this statement does not generalize. For example, a quadratic over the quaternions can have 1, 2, or infinitely many solutions. Dr. Robert Wilson and I looked at the number of solutions to a quadratic equations over $M_2(H)$, that is, 2 by 2 matrices over the quaternions. We used an injective homomorphism from $M_2(H)$ into the set of 4 by 4 matrices over the complex numbers to prove that almost all of the time - on a Zariski open set - there are at most 6 solutions.

Rachael Osborn, Pittsburg State University
Chromatic Number of Intersecting Subgraphs

In this presentation we will outline a solution to Problem # 11625 from the MAA Monthly. We will outline a proof for a formula involving the maximum chromatic number $\chi(G)$ of intersecting subgraphs of simple graphs on n vertices. Let G and H be subgraphs satisfy some technical conditions, we find and prove the maximum $\chi(G) + \chi(H) - \chi(G \cup H)$ when $G \cup H$ has n vertices. We will also discuss a lower limit for this number. This topic applies to map coloring and uses basis concepts in graph theory.

JoFawna Reali, California State University, Stanislaus
Susana Urquizo, California State University, Stanislaus
Calculus of Variations: Minimizing a Family of Functions

In Calculus of Variations, the Minimal Surface and Brachistochrone problems can be related to a family of functions, called a functional. Optimal solutions are the functions that minimize the functional. The theoretical solution to each of these problems can be found with methods of Calculus of Variations. In other words, we will derive the Euler-Lagrange equation from the Gateaux Variation of the functional set equal to zero, and the resulting partial differential equation will describe the true solution. The objective of this research is to numerically calculate the solution by selecting data points near the true solution and running them through the steepest descent algorithm until the points are aligned with the true solution. The curve that is mathematically described is the theoretical solution. This method of numerically calculating the minimum of a functional can be applied to anisotropic smoothing models in image processing such as restoring regions of interest in medical images.

Camila Reyes, California State University, Stanislaus
The Effect of Experiment Design on Network Inference

The network inference or reverse engineering problem consists on estimating the connectivity of a system from data. This problem is an important aspect of many areas of applied mathematics, specially when modeling biological systems. Recently, algorithms to reverse engineer Boolean network models have been developed; however, methods for collecting the data sets have not been examined sufficiently and it is not known what is the best way to collect data in order to obtain the best inferred network. In this talk, we show that for Boolean networks there are optimal ways to collect data for the network inference problem, based on the number of nodes in the network and the amount of data that is available. The results of our work provide a basis for researchers to obtain the most efficient data set, depending on experimental circumstances, to perform network inference.

Megan Rodriguez, Hood College
see **Natalie Guerra**

Amanda Russo, La Salle University
Perfect Graphs: An Application and a Conjecture

A perfect graph, G , is a graph for which the chromatic number of every induced subgraph H is equal to the clique number of H . The chromatic number of any graph is the minimum number of colors needed to color the vertices of G such that no two adjacent vertices share a color. The clique number of any graph, G , is the largest complete subgraph of G . In 1960, a Strong Perfect Graph Conjecture was presented by Berge and in 2002, it was confirmed by Chudnovsky, et al. resulting in the Strong Perfect Graph Theorem. This work considers both a concept and a problem that have come out of this theorem. The concept lies in the use of strongly perfect graphs and their application to optimal choosing. The problem that this research looks at is an outstanding open question that involves uniquely colorable perfect graphs and the connection these graphs have to clique number. This work shows the progress made on these topics and the conclusions that have been drawn from the research.

Alison Schuetz, Hood College
The Mandelbrot Set

The Multibrot Set \mathcal{M}_d is the set of all complex parameters $x \in \mathbb{M}$ for which the orbit of 0 under the map

$$f_{x,d}(z) = z^d + x$$

remains bounded. In particular, for $d = 2$, this set is the Mandelbrot set \mathcal{M} .

Of particular interest in complex dynamics is the orbit of 0 under $f_{x,d}$. We construct formulas for the coefficients of the power series in x of the infinitely iterated Multibrot polynomial

$$f_d^\infty(x) = \lim_{n \rightarrow \infty} f_d^n(x),$$

where $f_d^n(x)$ is defined recursively by the formula $f_d^0(x) = 0$ and $f_d^n(x) = (f_d^{n-1}(x))^d + x$ for $n \geq 1$.

To locate parameters $x \in \mathbb{C}$ which have orbits of period n , we compute the complex roots of polynomials $f_x^n(0)$. We sum the power series $f_d^\infty(x)$ and use these sums to locate the d fixed points of our function $f_{x,d}(0)$.

Katie Sopczynski, Slippery Rock University
Dowker Notations of Knots

In the field of knot theory, Dowker notation is an easy way to describe a given projection of a knot. Our talk will explore several questions concerning Dowker notation including: Why is there always an even and an odd number at each crossing? Why is Dowker notation a practical way to describe a knot? Is it true that for an n -crossing knot in the standard projection, there are always $2n$ different notations?

Bridget Toomey, Grinnell College
Incongruent Restricted Disjoint Covering Systems (IRDCS)

It is impossible for a covering of the integers to be both incongruent and disjoint; however, systems of congruences that cover only finite intervals can satisfy both of these conditions. These systems are called incongruent restricted disjoint covering systems (IRDCS). We place a particular emphasis on our findings regarding a special case of IRDCS, the $9 - 6 - 3$ construction. We also present modifications we made to an existing program which computes all IRDCS of a given length in order to find only $9 - 6 - 3$ constructions.

Whitney Turner, Freed-Hardeman University
see **Lauren Akin**

Susana Urquizo, California State University, Stanislaus
see **JoFawna Reali**

Tara Walker, Seattle Pacific University
see **Stephanie Carter**

Catherine Watkins, Johns Hopkins University
Optimal Digital Elevation Model Estimation

In this research, we explore techniques in digital elevation model generation without the use of fiducial points. A digital elevation model (DEM) is a three-dimensional representation of a terrain generated from a set of two-dimensional images. The current method of DEM formation consists of two steps: registration and DEM generation. In this talk we formulate a one-dimensional strategy, whose basis is a minimization problem, which can be used to generate a DEM without the use of fiducial points. The method we developed makes use of a variety of cost functions, which determine the similarity between photographs, and various deterministic and stochastic minimization techniques, which minimize the costs functions and give us the unknown DEM parameters, simultaneously.

Laura White, Arkansas State University
Construction of Generalized Minimum Aberration Designs from Handmard Matrices of size 32

Regular fractional factorial designs are widely used experimental designs for studying effects of two or more variables simultaneously, but leave large gaps in run size. Non-regular fractional factorial designs can be constructed for every run size that is a multiple of four, which allows run size flexibility and economy. My research will focus on construction of optimal designs from Handmard Matrices of size 32 using graphic processing unit (GPU) technology, with a primary objective of providing, for the first time, comprehensive tables for the best 32 runs designs available. Creating design tables make it possible for engineers and scientist to plan experiments for any number of variables less than 32 to be studied within 32 runs.

Sarah White, Wartburg College
see **Stephanie Carter**

Anna-Rose Wolff, George Mason University
Support Vector Machines and the Exterior Point Method

Support Vector Machine (SVM) learns an unknown function by recognizing underlying patterns within large data sets. One of the goals of the SVM is to find the support vectors, or data points that contain the most important information about the unknown function. We develop an optimization algorithm for finding the support vectors based on an exterior-point method and data decomposition.

Leah Wright, Freed-Hardeman University
see **Lauren Akin**