The Twenty-First Annual Nebraska Conference for Undergraduate Women in Mathematics

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TALK ABSTRACTS

PLENARY TALKS

Dr. Christine Darden
NASA Langley Research Center
Faster than the Speed of Sound

Hired by NASA in June 1967 as a Human Computer, Dr. Christine Darden will discuss briefly her work as a Computer in the Re-Entry Physics Branch while NASA was preparing for Man to walk on the moon. Darden will then discuss how she was transferred to an Engineering Office to begin looking at the possibility of reducing the noise level of a sonic boom. She will discuss her development of two computer codes and the experimental research that accompanied her work. Darden will culminate her talk with the current status of work in supersonics that is a follow-on to her 25-year career in research.

Dr. Zhilan Feng
Professor of Mathematics
Purdue University
Applications of Mathematical Models in Biology and Epidemiology

Mathematical modeling of infectious diseases has affected disease control policy throughout the developed world. Policy goals vary with disease and setting, but preventing outbreaks is common. In this talk, I will present several examples to demonstrate how various models can be used to answer questions related to disease control and prevention for specific diseases in real populations. These models are systems of differential equations. The mathematical results are motivated to address specific biological questions. Models and applications to ecological systems will also be discussed.

Dr. Margaret Holen
Lecturer
Princeton University
Math, Finance, and Decisions

Financial choices affect us every day – in how we collect our pay and purchase daily necessities, how we invest and plan for the future, pay for education and housing, and protect ourselves through insurance for our cars. In turn, the companies that provide us with those services navigate challenges of delivering complex products, at fair and reasonable prices, while protecting from fraud. In recent years, technology has created novel ways for services to be delivered. Some examples of "FinTech" products include Acorn for savings, Betterment for investing, and Zelle for payments, among thousands of others.

FinTech brings opportunities for mathematicians to shape how financial resources are allocated. Efficiency requires material decisions to be made at scale, using models and systematic methods in place of ad-hoc judgments. Those models are grounded in math-finance concepts, including time-value-of-money and the risk of loss and benefits of diversification, which can be expressed with differential equations and probability toolkits. Machine learning becomes increasingly important as decision points occur at higher frequencies and over larger numbers of choices, from which loan application to fund or which securities to buy. Optimization is central to building those models and to the core goal of identifying the best choices.

Financial decisions can have significant impact – Whose company will get launched? Who will be denied a loan? Even amid the impressive progress, examples of flawed decisions abound: examples from markets misallocating capital and from my own career path. Those illustrate how complex and error-prone decision processes of all types can be. However, systematic approaches can enable faster improvement, and mathematicians bring valuable skills and knowledge to drive change. I look forward to sharing reflections on opportunities in contemporary finance and on decisions that led me here, including my decision to study math, to pursue a career in finance, and most recently to combine those as a lecturer at Princeton and an investor and advisor to early-stage companies.

Talks by Undergraduate Students

Allegra Allgeier, Kalamazoo College
On the Construction of a Convex Ideal Polyhedron in Hyperbolic 3-Space

The purpose of this project is to construct a convex ideal polyhedron in hyperbolic 3-space whose Poincaré dual satisfies the conditions in the main theorem of Rivin's paper, "A Characterization of Ideal Polyhedra in Hyperbolic 3-Space." The project focuses on platonic solids and in particular, a thorough construction is given for hyperbolic cubes and octahedrons.

Sydney Benson, University of St. Thomas
Jessica Mohr, University of St. Thomas
Modeling Economic Capital Using Market and Mortality Risk and Copula Methodology

The proper estimation of economic capital (EC), a combination of investment loss and insurance liabilities minus capital, is essential to life insurance companies to ensure they can pay claims as needed. Alternatively, the outcome of inaccurate predictions is exemplified by the financial crisis of 2008. EC is difficult to estimate due to the multiple dependent risk factors involved in its calculation; however, multivariate probability distributions with uniform marginal distributions, or copula models, make it possible to address this interdependence, particularly in a low interest environment. The novelty of this project lies in the inclusion of both market and mortality risks and the interdependence of the two in a single model. A validated, practically applicable model of EC, using this framework, has been provided to insurance companies utilizing R, an open source software. This model takes into account the varied portfolio and monthly loss history of the company to provide a yearly EC estimate.

Brittany Bianco, Metropolitan State University of Denver
Leigh Foster, Metropolitan State University of Denver
Automorphisms and Characters of Finite Groups

Representations are special functions on groups that allow us to study abstract finite groups using matrices over the complex numbers. Much of the information about representations can be understood by instead studying the trace of the matrices, giving what we call a character. It turns out that various types of automorphisms (like those of the original group itself and those of any field containing the values of the characters) naturally permute the set of characters of the group. We analyze this phenomenon for a finite matrix group called $\mathrm{Sp}(4, q)$ (the "symplectic group of degree 4") in the context of some important current conjectures in the field of character theory.

Rebecca Broschat, Northern Arizona University
Alyssa Stenberg, Northern Arizona University
A Geometric Approach to Generalized Frobenius Numbers

Given a set $S = \{a_1, a_2, \ldots, a_n\}$ of relatively prime positive integers, the kth Frobenius number, $g_k(S)$, is the largest natural number that can be expressed as a linear combination of $\{a_1, a_2, \ldots, a_n\}$ over the nonnegative integers in precisely k distinct ways. We will present new results on computing $g_k(S)$ by computing integer lattice points inside an associated $n-1$ dimensional polytope.

Alanis Chew, Youngstown State University
Madeline Cope, Youngstown State University
Come Converge: Let's Talk About Clustering

Data clustering is the unsupervised classification of patterns into groups or clusters that is based on recognizing the similarities or dissimilarities between the data. It can be used in a wide variety of disciplines, ranging from astrophysics to zoology. The two main branches of data clustering are hierarchical and partitional. This talk will focus on two techniques, Mutual Nearest Neighbor, a hierarchical clustering technique, and Spectral Clustering, a partitional clustering technique. Mutual Nearest Neighbor is an algorithm that merges the least dissimilar mutual pair, and Spectral Clustering is a graph theoretic partitioning that uses eigenvalues of the graph for dimensionality reduction. These data clustering techniques are two of many that can be used to analyze a wide range of data sets. This work was made possible by generous support from the J. Douglas and Barbara T. Faires Center for Undergraduate Research in Mathematics.

Patricia Commins, Carleton College
Recovering Conductances of Resistor Networks in a Punctured Disk

The response matrix of a resistor network is the linear map from the potential at the boundary vertices to the net current at the boundary vertices. For circular planar resistor networks, Curtis, Ingerman, and Morrow have given a necessary and sufficient condition for recovering the conductance of each edge in the network uniquely from the response matrix using local moves and medial graphs. We generalize their results for resistor networks on a punctured disk. We discuss additional local moves that occur in our setting, prove several results about medial graphs of resistor networks on a punctured disk, and define the notion of z-sequences for such graphs. We also define certain circular planar graphs that are electrically equivalent to standard graphs and turn them into networks on a punctured disk by adding a boundary vertex in the middle. We prove such networks are recoverable and generalize this result to a much broader family of networks.

Madeline Cope, Youngstown State University
see **Alanis Chew**

Mayleen Cortez, California State University, Channel Islands
Using Time-Dependent Sensitivity Analysis to Combat Tuberculosis

Although many organizations throughout the world have worked tirelessly to control tuberculosis (TB) epidemics, no country has yet been able to eradicate the disease completely. In this talk, we present two compartmental models representing the spread of a TB epidemic throughout a population. The first is a general TB model; the second is an adaptation for regions in which HIV is prevalent, accounting for the effects of TB/HIV co-infection. Using active subspaces, we conduct time-dependent sensitivity analysis on both models to explore the significance of certain parameters with respect to the spread of TB. We use the results of this sensitivity analysis to determine the most effective strategies for treatment and prevention throughout the epidemic.

Jane Cox, Brigham Young University
Jordan Spencer, Brigham Young University
Survey Data and Mathematical Modeling in Prioritizing Water Interventions in Developing Countries

We describe a method for combining the World Health Organization's cost-effectiveness analysis with countrywide survey data in order to construct an ordered ranking of the areas within a given country that have the highest need for a more reliable water source; it also includes the optimal method of water intervention. We address a key problem in the charitable water sector: While survey data is available, due to privacy issues, much of the geographical and spatial data is lost or confounded. This loss disconnects the information from the locations in which they were found, making the data largely unusable. To overcome this issue, we propose using a combination of Voronoi modelling and gamma distributions to estimate an accurate representation of the data, allowing charities to overcome the lost information and increase their ability to use the available data. This method has been tested on the countries of Namibia and Angola and could be expanded to many more.

Marianne DeBrito, Lawrence Technological University
Annaliese Keiser, Bowling Green State University
Efficiency of a Moving Mesh System with a Curvature-type Monitor Applied to Burgers' Equation

Moving Mesh Methods are adaptive techniques to approximate solutions to partial differential equations numerically. A moving mesh system consists of a discretized physical PDE that evolves in time together with a PDE that adapts the discretization mesh using a monitor function. The moving mesh PDE is viewed as a mapping from a computational to physical domain. In this project, we explore properties of the moving mesh system when the physical solution has a steep gradient and large curvature, depending on parameter ϵ, over a small interval in the domain. Using a curvature-type monitor, we prove an explicit dependence of the derivatives of mappings between the computational and physical domains on ϵ. In addition, we show a similar dependence for the mesh spacing, which is important in quantifying discretization errors. These results are verified numerically for a known physical solution. Numerical evidence also suggests a significant reduction in steep gradients when using moving mesh with this type of monitor. These estimates show an explicit reduction of the number of equations needed to approximate the physical PDE with the moving mesh. We can further control mesh spacing and steepness of derivatives by adjusting parameters in the monitor function. As an application, we use our moving mesh system to model Burgers' Equation, which satisfies the hypothesis of our theorem.

Leigh Foster, Metropolitan State University of Denver
see **Brittany Bianco**

Yuheng Fu, Washington University in St. Louis
Simulation Study of Bulk Reference-Based Imputation Method for Single-Cell RNA-seq Data

Single-cell RNA sequencing (scRNA-seq) enables transcriptome profiling of individual cells. However, scRNA-seq data is complicated by the dropout effect, excess zero or near zero data counts. Here, we propose a computational method to impute dropouts in scRNA-seq data under semi-supervised settings. Based on our simulation results, we can accurately estimate cell type composition with quadratic programming and our weighted average formula. Dropouts can be then identified by integrated network adjacency ranking and be imputed with a matrix completion algorithm. Our statistical test demonstrates that identifying dropouts before imputation significantly enhances accuracy for matrix completion. This method will benefit researchers by providing putative expression data for gene markers that are crucial for biomedical studies (e.g., the study of cell identity and behavior), but are prone to dropout. Future work would focus on continuing to improve the robustness and efficacy of the method by building a more dynamic model.

Juliann Geraci, State University of New York at Oswego
The Hidden Information in Infinite Series Arising from Graphs

Letting h_n denote the number of walks of length n in a directed graph G, we study the infinite sequence of numbers $h = (h_0, h_1, h_2, \dots)$, and how properties of G are encoded in its growth. To extract information from h, we will study its generating function $H(t) = \sum_{n \geq 0} h_n t^n$. We will show that $H(t)$ coincides with a rational function $f(t)$, and that properties of G (e.g., the number and lengths of oriented cycles in G) influence the form of $f(t)$.

Audrey Goodnight, Agnes Scott College
Laura Stordy, Agnes Scott College
Braces and their Opposites

Braces are a relatively new construction, created to find solutions to the Yang-Baxter equation. A left skew brace is a set B with two binary operations, $\cdot$ and $\circ$ such that $(B,\cdot)$ and $(B,\circ)$ are groups, and for all $a,b,c \in B$; $a \circ (bc) = (a \circ b)a^{-1}(a \circ c)$. The Truman opposite of a brace B, denoted B^{op}, is defined by $(B,\cdot,\circ')$, where $a \circ' b = (a^{-1} \circ b^{-1})^{-1}$. We give instances where $B \ncong B^{\text{op}}$.

Mary Greene, Washburn University
Game Design Course

We develop the curriculum for a game design class, as well as write the source to be used by the students in attendance. Our plan of study includes exposure to the variety of game genres, many game mechanics, and tools to analyze these. Substantial use of case studies through the application of mathematical and statistical principles to analyze existing games that clearly illustrate these mechanics will constitute a central part of the course. Through these we describe many ways that combinatorics, matrices, game theory, graph theory, and light applications from calculus are present in the design of these games. The course is designed to have minimal prerequisite mathematical background, but by the end of the term, students will have acquired an arsenal of tools with which to develop their own games—as well as recognize where these designs fit in the broad variety of game genres and mechanics. The course ends with each student creating their own game and critiquing its qualities, cataloging its genre and mechanics, and analyzing it for balance and mechanical structure. An emphasis in the presentation will be placed on describing how we chose the most essential elements of mathematics that best serve budding designers at this level.

Asimina Hamakiotes, Macaulay Honors at Baruch College
Eta-Quotients of Prime or Semiprime Level and Elliptic Curves

In this talk, we investigate questions related to eta-quotients. We prove that eta-quotients that satisfy a condition of Newman are always modular forms if N is coprime to 6, and study the case of all odd levels. We also prove that all eta-quotients which are modular forms for $?_1(N)$ are also modular forms for $?_0(N)$. We found a condition for the existence of an eta-quotient as a modular form for prime levels, and generalized this for semiprime levels as well, which has improved upon recent work of Arnold-Roksandich, James, and Keaton. In addition, we investigate representing modular forms associated to elliptic curves in terms of linear combinations of eta-quotients, and provide some new examples.

Nicole Harris, Denison University
Alexandra Tubbs, Denison University
Algorithmic Investigation of Substitution Tilings and their Associated Graph Laplacians

A 2-dimensional tiling is formed from a collection of shapes put together without any gaps or overlaps. We often associate a graph to a tiling with a node for each tile and an edge between nodes whose tiles share a side. Our research deals with two aperiodic tilings, the chair and the pinwheel. We introduce an algorithm for computing graph Laplacians associated with these two examples, as well as explain the significance of such matrices. In examining the algorithm we consider the fact that each iteration contains copies of the previous iteration with some additional edges. We discuss how these additional edges are identified using the inflate-and-subdivide rule for the tiling.

Rainie Heck, Oberlin College
From Ideal Polyhedra to Fundamental Domains in Hyperbolic 3-Space

Our research goal is to better understand the relationship between the polyhedron and the group associated with a fundamental domain in hyperbolic 3-space. In this talk, we will study torsion-free groups and determine a formula for how many edge classes a given abstract polyhedron must have. We will use that result to classify all fundamental domains on the cube with torsion-free groups, including a discussion of the explicit groups associated to those domains. We will then turn to more general fundamental domains and prove a series of results about how properties of the group place restrictions on the edge classes in the quotient manifold. These results give insight into how the polyhedron and the group associated to a fundamental domain interact, as well as offer concrete tools to find fundamental domains.

Paige Helms, University of California, Riverside
Using Origamis to Examine the Action of $SL(2,\mathbb{Z})$ on Quadratic Differentials

The purpose of this research is to establish a lower bound for the number of orbits of the $\mathrm{SL}(2,\mathbb{Z})$ action on the space of quadratic differentials of a genus g surface, which can be identified with the cotangent bundle to Teichmüller space T_g. We accomplish this through an algebraic interpretation of a pair of minimally intersecting curves that fill a surface S_g of genus g, where for $g > 2$, the minimal intersection number is $i(a,b) = 2g-1$. Such a pair of curves can be visualized on a square tiling of a surface S_g, carrying the structure of an origami. We call the Minimally Intersecting, Filling Pair (a,b) on this Origami a MIFPO. This interpretation gives us a way to examine the action of $\mathrm{SL}(2,\mathbb{Z})$ on a given surface by calculating its monodromy group, whose type gives us a lower bound for the number of orbits. So far, we have shown the existence of one orbit in $n=5$, four orbits in $n=7$, three orbits in $n=9$, and at least two orbits in $n=11$.

BethAnna Jones, State University of New York at Geneseo
Amanda Stanley, Grand Valley State University
Tracking Neural Activity: Automated Image Analysis

Modern optical methods such as two-photon imaging of calcium fluorescence allow us to view the activity of tens of thousands of neurons in the brain. Manual methods of identification and analysis of individual neurons in these images is tedious. Automated techniques for research exist, but as shown by the 2017 SURIEM project, they find only roughly half the cells and disagree on half of the results. We aim to build on this work and improve automatic analysis techniques of neural images. Leading methods analyze neural images using singular value decomposition and constrained non-negative matrix factorizations. We will compare the cells identified with these methods and improve them by analyzing fluorescence patterns of the cells, investigating correlation patterns across cells, and aligning cells between sessions to compare cell activity across days as animals learn. This work was completed as part of the 2018 SURIEM program supported by Michigan State University, the NSA, and the NSF.

Annaliese Keiser, Bowling Green State University
see **Marianne DeBrito**

Haley Knox, Eastern Connecticut State University
The Spread of Information on Twitter Based on Sentiment

Twitter is a social media platform where users can send messages, or tweets, in real time and other users can retweet, quote or reply to spread the tweet. This makes Twitter one of the fastest ways to spread information. In this research, we analyze the sentiment of a tweet and how it disperses on Twitter. We classify the sentiment of randomly selected tweets as positive, negative, or neutral using R. In addition, we identify features of tweets that evoke similar emotional responses, such as patterns between sentiment and the number of retweets. To visualize the spread of a tweet, we create response networks with nodes being users that responded to the tweet and links signifying that one user follows the other. Our network construction better represents from whom a user in the network saw the tweet compared to links indicating whom a user retweeted. We build response networks for a sample of positive and negative tweets and analyze them using graph theoretic measures to compare features of the network and the corresponding sentiment.

Emma Kuntz, Furman University
Modeling Bacteria Growth and Antibiotic Interaction with Cellular Automata

I have made a cellular automata model to show the growth of bacteria and its interaction with antibiotics. The model uses stochastic rules to model the probability of a bacteria being resistant to a given antibiotic (out of 10 total antibiotics present in the model). The model incorporates the Disk Diffusion Method to compare the resistance of bacteria to many different types of antibiotics. Differences in bacteria types for the given antibiotics are also compared in the model. The goal of this cellular automaton model is to not only serve as a visual representation of the interaction between bacteria and antibiotics, but also be much more efficient than performing tests in the lab in terms of both time and money.

Alicia Ledesma Alonso, Grinnell College
Hongyuan Zhang, Grinnell College
Artworks and Articles Meet Mapper and Persistent Homology

Since its recent birth, topological data analysis has proven to be a very useful tool when studying large and complex datasets. In this talk, we present how we use persistent homology tools and Mapper to explore two datasets from the following sources: Metropolitan Museum of Art (MET) and arXiv. After learning the preliminary theory of topology, the two datasets were studied independently. For the MET project, we use Mapper to guide feature selection in building a logistic regression model. We then use persistent homology to help differentiate between two subsets of artworks. For the arXiv project, we use persistent homology to derive a general sense of the shape of the data. We next use Mapper to further explore the relationships discovered from the persistent homology results by analyzing trends and certain features in the Mapper visualizations. By using these tools, detailed insights are given in understanding the complexity of each dataset.

Xinru Liu, Wheaton College
Multimodal Data Fusion in 3D Printing Quality Prediction

Additive manufacturing, or three-dimensional (3D) printing, is a promising technology that enables the direct fabrication of complex shapes and eliminates the waste associated with traditional manufacturing. A major issue that adversely affects its performance, and hence wider adoption, is that material solidification in the printing process yields geometric shape deviation. This talk presents a data-driven approach to predict the quality of 3D printed objects using multiple measurement image data sources with different accuracy and measurement efficiency. Dimension reduction techniques are employed for extracting features from measurement data and quality metrics are defined using clustering algorithm. We propose a two-level random forest classification model trained with printing input parameters and extracted features from the two sensors respectively at each step to predict quality. Such methods could guarantee time efficiency while maintaining high accuracy. The result shows feature extraction from high-dimensional measurement data as a promising technique to automate and optimize the 3D printing process.

Sarah Lubow, Loyola University New Orleans
Carlie Triplitt, University of Science and Arts of Oklahoma
Vertex-Minimal Planar Graphs with a Prescribed Automorphism Group

In 1939, Frucht proved that for any finite group G, there exists a graph Γ such that the automorphism group of Γ is isomorphic to G. Naturally, this result gave rise to numerous extremal problems in graph theory. For instance, vertex-minimal graphs with a prescribed automorphism group are the subject of prior research by numerous authors. In this talk, we will discuss our proof of a conjecture made in 1980 by Marusic on the order of vertex-minimal planar graphs with cyclic symmetry of even order. Our proof completes a theorem giving the order of all vertex-minimal planar graphs with cyclic automorphism groups. We will also discuss further our proof regarding the order of vertex-minimal planar graphs with dihedral symmetry. This work was completed as part of the REU program at University of Texas at Tyler.

Meraiah Martinez, Benedictine College
Pattern Avoidance in Acyclic Digraphs

A pattern of length k is a permutation in S_k. A permutation $\pi \in S_n$ avoids the pattern σ if no subsequence of length k has elements in the same relative order as σ. Our research extends this concept to directed acyclic graphs. A directed acyclic graph G avoids the pattern if there are no directed paths whose vertices contain a subsequence in the same relative order as the pattern. For certain sets of length 3 patterns, we find the number of directed acyclic graphs on n vertices that avoid all of the patterns in the given set. Furthermore, for certain sets of patterns where exact enumerations have not been found, we provide a comparison to the number of graphs avoiding a different set of patterns. This work was completed as part of the REU program at the University of Texas at Tyler.

Sofia Martinez, University of California, Riverside
Chromatic Symmetric Functions with respect to Complete Graphs

Properties of chromatic symmetric functions for specific graph classes have long been studied. One of the fundamental questions is whether a chromatic symmetric function uniquely determines a tree. This question was posed first by Stanley in 1995 and it remains an open problem, although it has been answered in the affirmative for a number of special classes of trees including caterpillars and spiders. Here we show the result holds for generalized spiders (i.e. line graphs of spiders) too, thereby extending the work of Martin, Morin, and Wagner. A second fundamental question is whether a chromatic symmetric function is e-positive. Here, in this presentation, we establish that certain classes of generalized spiders (those known as generalized nets) are not e-positive. This is joint work with Angele M. Foley, Joshua Kazdan, Larissa Kroell, Oleksii Melnyk, and Alexander Tenebaum.

Bethany Matsick, Liberty University
Twisted Hermitian Codes in the McEliece Cryptosystem

In 1978, Robert McEliece introduced a public key cryptosystem based on the difficult problem of decoding a random linear code. Due to its large key size, the McEliece cryptosystem has yet to see widespread use. However, given that its security does not rely on factorization as the commonly employed RSA and elliptic curve cryptosystems do, it is now being considered as a candidate for post-quantum cryptography. To ensure that a code appears random in this system, we desire a code with a Schur square that behaves like that of a random linear code, meaning the dimension of its Schur square is equal to that of a random linear code of the same dimension. Because most classical families of codes fall far short of this ideal, we develop a family of "twisted" Hermitian codes with a Schur square dimension comparable to that of random linear code. We show that, when constructed properly, these "twisted" Hermitian codes not only achieve a high dimensional Schur square but also maintain a reasonable data transfer rate. The twisted construction is a variant of that considered by Peter Beelen, Martin Bossert, Sven Puchinger, and Johan Rosenkilde. This is joint work with Austin Allen, Keller Blackwell, Olivia Fiol, Rutuja Kshirsagar, and Zoe Nelson, supervised by Gretchen Matthews.

Amanda McAdams, Washington University in St. Louis
Cellular-scale Modeling of Oncogenic Proteins

Mutations in the RAS family of proteins have been implicated in roughly 25% of all human tumors and up to 90% in certain types of cancerous tumors, such as pancreatic cancer. RAS mutations can lead to overactive signaling in cells, which prevents cell death and leads to tumor growth. In order to better understand the dynamics of RAS protein interactions with the cell membrane and other proteins (specifically RAF proteins), a combination of atomistic data and a continuum scale model is constructed. The various interactions are incorporated into the model through a free energy functional, which describes the available work in this thermodynamic system. Furthermore, the evolution equations describing the changes in the membrane's lipid concentrations, the membrane's height, and the proteins' movement are derived using dynamic density functional theory and Langevin dynamics.

Breanna McBean, California State University, Fullerton
The Shape of Large Soap Bubbles

The behavior of bubbles has intrigued both physicists and mathematicians for over a hundred years. The intrigue comes from the inherent ability of a soap-bubble to solve the well-posed mathematical problem: Given a bounded volume, what is the shape of the boundary surface with minimal area? In this classical problem, which was first formulated by nineteenth-century physicist Joseph Plateau, the weight of the soap film is neglected. However, as the size of the bubble grows, macro-scale effects such as gravity, become more important and the shape of the surface can change significantly. We seek to characterize these qualitative changes via the following modified problem that captures the behavior of both small and large soap bubbles: Find the surface that minimizes the sum of surface and gravitational energy and encloses a specified volume. Additionally, this question is intimately related to the construction of heavy surfaces and provides practical insights into the shapes of energetically optimal domes.

Stephanie McCoy, Northern Arizona University
Convex Hull Achievement Games in Euclidean Space

We study a game where two players take turns choosing elements from a fixed finite set of points in $\mathbb{R}^n$ until the convex hull of the jointly selected elements contains all the points of a given winning set. The winner of the game is the last player who was able to make a move. We determine the nim number of these games for several configurations of points, including one-dimensional games and all games with a winning set consisting of vertex points. This allows us to determine the outcome and the optimal strategy of these games.

Morgan Mitchell, California State University, Sacramento
When and How to Use Math Based Card Tricks in the Classroom

In this talk we compile examples of math-based magic tricks for educators to use in their classroom, and survey research showing that such tricks can help teach and engage students. There is not much overlap between developers or math-based tricks and education researchers—this work helps close that gap. Examples of tricks that have been found in various books and on the internet will be used as examples, and we fully explain the math in the tricks and suggest math course topics when they can be used. We then suggest ways to present these tricks by utilizing education research-based techniques on how to engage students.

Jessica Mohr, University of St. Thomas
see **Sydney Benson**

Emily Montelius, Coe College
Fault-Free Tileability of Rectangles, Cylinders, Tori, and Möbius Strips with Dominoes

We study fault-free tileability of boards with dominoes as tiles, where the boards are rectangles, cylinders, tori, and Möbius strips. A tiling is a way of arranging pieces on a board, such that there is no space left uncovered, nor any space covered by more than one tile. To be fault-free every line that intersects the tiling must also intersect the interior of at least one of the tiles. We have complete results for cylinders and tori.

Rachel Morris, University of Richmond
Ineffective Sets and the Region Crossing Change Operation

Region crossing change (RCC) is an operation performed on a selected region of a link diagram that reverses all crossings incident to that region. A set of regions is ineffective if, after RCC moves are performed on each region in the set, the resulting link diagram has the same crossing information as the original one. Ineffective sets are key to understanding how many RCCs it takes to transform one diagram to the other. In this talk, we characterize the ineffective sets for all knots and reduced link diagrams. These are determined by checkerboard shading and a variant of checkerboard shading called tri-coloring. As an application, we will determine the maximum number of RCC moves needed to transform one knot diagram to another diagram with the same underlying projection.

Sophia Perron, University of Minnesota, Twin Cities
Comparing Alternative Metrics of Metapopulation 'Patch Value' to Identify Effective Marine Protected

In marine conservation, there is a recognition that spatial connectivity information should be used to inform marine protected areas (MPAs) placement and several potential ways to use that information, but no consensus about which is the most effective. We compared hypothetical MPA networks created using different design criteria to determine which design strategy produces MPA networks best achieve performance goals. Using an existing set of population models of two oceanographically distinct regions of the California coast, we evaluated the utility of different metrics of patch value (including habitat quality and quantities related to larval connectivity: self-persistence, total export, total import, deletion, and centrality) for MPA planning. The MPA planning strategies were compared using a spatially explicit, age-structured population dynamic model with density-dependent recruit survival for 4-5 species in each of the two coastal regions. Larval connectivity between habitat patches was derived from numerical ocean modeling simulations. Each hypothetical network for each region was simulated by protecting habitat patches that have high scores in one of the patch value metrics. By simulating population dynamics using each hypothetical network, it was discovered that self-recruitment is very important in areas which dispersal has strong directionality. In regions with diffusive dispersal patters, habitat is the best option for MPA placement.

Giulia Pintea, Simmons University
Using Quality-of-Scores to Guide Prostate Radiation Therapy Dosing

Since prostate cancer patients have high survival rates, an important factor in treatment is to avoid degradation in quality-of-life during and after treatment. The connection between the radiation a patient receives and his reported side effects has not been quantitatively analyzed. We use deep learning algorithms and statistical models to explore this relationship. We use interpolation methods to generate more data in order to leverage transfer learning. Using augmented data, we train a convolutional autoencoder network to obtain near-optimal starting points for weights of our final convolutional neural network (CNN). Our CNN analyzes the relationship between patient-reported quality-of-life and radiation dosage in the bladder and rectum. We also use analysis of variance and logistic regression to explore organ sensitivity to radiation and develop dosage thresholds for each organ region. Our findings show a connection between rectal radiation dosage and changes in quality-of-life. We identify regions of both the bladder and rectum that are highly correlated with changes in individual patient symptoms. Finally, we estimate radiation therapy dosage thresholds for the rectum to determine how high radiation therapy dosage needs to be in order to trigger collateral symptoms.

Kaleigh Roach, North Greenville University
Lee Trent, Rose-Hulman Institute of Technology
Simulating and Modeling Influence in Referendum Elections with Graph Theory

In referendum elections, voters are often required to cast simultaneous votes on multiple proposals. A voter's opinion on one proposal can depend on the outcome of another, and that dependency can lead to undesirable or even paradoxical outcomes. This problem is called the separability problem. In this talk, we will introduce certain types of digraphs called influence diagrams, which are derived from the core preferences of a voter, to model the influence relationship between proposals and sets of proposals, as well as explore theoretical results about the structure of those digraphs. Using those digraphs, we can mitigate the undesirable results of the separability problem by sequencing elections more effectively, yielding election results that better reflect electorate preferences. We will discuss this application and the simulation of this model in election sequencing.

Rosa Rossi-Goldthorpe, Bowdoin College
Flow-Kick Systems in Neuroscience: Exploring the Treatment of Bipolar Disorder

Bipolar disorder is a common yet puzzling psychiatric illness, and the precise biological mechanisms for the most common treatment, lithium, are still unknown. One hypothesized cause of mania in bipolar patients is abnormal calcium signaling in neurons, leading to a significant increase in neurotransmitter release. To better understand how lithium affects calcium dynamics to stabilize mood, we develop a "flow-kick" model of a neuron subject to lithium treatment. A flow-kick model takes a continuous dynamical system and applies a repeated, discrete perturbation to the system. We use a Morris-Lecar model to represent neural bursting, and apply repeated perturbations representing regular doses of lithium to the neuron. We analyze the dynamics of the flow-kick model to explore how neuron firing and neurotransmitter release can be controlled by pharmaceutical treatments. Long-term goals include determining the optimal lithium dosage to maintain a "neurotypical" firing pattern and investigating the "resilience" of different model behaviors to treatment and/or environmental stressors.

Barbara Schweitzer, State University of New York at Geneseo
Spectral Characterizations of Anti-Regular Graphs

We studied the eigenvalues of the anti-regular graph on n vertices. Using these eigenvalues, different characteristics of graphs can be identified. The adjacency matrix of the anti-regular graph and Chebyshev polynomials led us to equations for the eigenvalues, allowing us to characterize the eigenvalues. The equations for the eigenvalues produced a strict bound on the eigenvalues values. The bound $\Omega = \left[\frac{-1-\sqrt{2}}{2}, \frac{-1+\sqrt{2}}{2}\right]$ contains no eigenvalues of the anti-regular graph on n vertices, except for the trivial eigenvalues, $\lambda_0 = 0$ or $\lambda_0 = -1$. Outside of the Ω interval, for large enough n, eigenvalues can be found. Interval bounds for each eigenvalue can be described, as well as a bound for the minimum and maximum eigenvalues. We can also describe the bipartite character of the graph, as it is nearly symmetric about $-\frac{1}{2}$. Finally, a connection between the anti-regular graph and threshold graphs can be described.

Laura Seaberg, Haverford College
Intersections of Shortest Taxicab Paths in the Sierpiński Carpet

In recent work, Berkove and Smith have developed an algorithm to construct shortest taxicab paths in the Sierpiński carpet and some of its higher-dimensional generalizations. We consider an extension of this problem examining minimal area surfaces bound by shortest taxicab paths in higher-dimensional fractals. Such a minimal surface will have zero area if and only if the associated shortest paths have non-empty common intersection. Specifically, we give a set of necessary and sufficient conditions on the relative positions for three points in the carpet which characterize when the pairwise shortest taxicab paths have non-empty triple intersection. Finally, we indicate how our work might generalize to higher dimensions.

Elizabeth Spaulding, University of Nebraska-Lincoln
Boundary Layer Separation in Turbulent Fluids

Wake vortex and turbulence generated by aircraft can cause serious danger to a trailing aircraft due to flow instability and uncontrollable vortices. There have been recent incidents when wake turbulence created by larger aircraft has resulted in a sudden loss of altitude to smaller trailing aircraft. The wake takes the form of counter-rotating vortices, trailing from the tips of the wings. In aerodynamics, these vortices are responsible for increasing drag. Computational fluid dynamics (CFD) simulations are being used to gain a broader understanding of this dangerous wake turbulence as well as to support the design of space launch and aircraft vehicles.

Studying wake turbulence and the Navier-Stokes equations both theoretically and computationally is the starting point for learning how to reduce drag to stabilize wake vortices. Detailed mathematical models of wake vortices have been developed, and flow separation leading to turbulent flow has been studied both mathematically and through CFD simulations. Boundary layer theory was used as a starting point for derivations of mathematical models of flow separation and development of numerical schemes. Data from in-house code developed using derivations of the Navier-Stokes equations with an emphasis on boundary layer theory is compared to well-established results in the field of fluid dynamics.

Jordan Spencer, Brigham Young University
see **Jane Cox**

Amanda Stanley, Grand Valley State University
see **BethAnna Jones**

Alyssa Stenberg, Northern Arizona University
see **Rebecca Broschat**

Suzanna Stephenson, Brigham Young University
Survey Data and Mathematical Modeling in Prioritizing Water Interventions in Developing Countries

We present a multivariate numerical rootfinding algorithm that finds all real zeros in a given compact region in C^n of a system of functions. Our method builds on the ideas of Nakatsukasa, Noferini, and Townsend of subdividing the original search interval and approximating the functions with Chebyshev polynomials. We then use a variant of the method of Telen and van Barel, finding the roots in each subdomain by computing eigenvectors of the Chebyshev form of certain Möller-Stetter matrices constructed with a well-chosen basis. We compare our algorithm, in terms of accuracy and speed, to other popular numerical rootfinding algorithms. In many instances, this algorithm outperforms all known competitors.

Laura Stordy, Agnes Scott College
see **Audrey Goodnight**

Sanah Suri, Grinnell College
One-Seventh Ellipse Problem

A curious mathematical phenomenon is called the One-Seveth Ellipse. Take the digits from the decimal expansion of $\frac{1}{7}$, namely 142857, and form six points in the plane: $(1, 4)$, $(4, 2)$, $(2, 8)$, $(8, 5)$, $(5, 7)$ and $(7, 1)$. The surprising fact is that these six points lie on an ellipse. Moreover, if we take consecutive digits from the decimal expansion two at a time, the six points $(14, 42)$, $(42, 28)$, $(28, 85)$, $(85, 57)$, $(57, 71)$ and $(71, 14)$ also lie on an ellipse. Ordinarily, an ellipse can be determined by five points so our research was motivated by why such a phenomenon always occurs with these six. On investigation, we found that the phenomenon can be extended to all conic sections and particular sequences of six digits. This talk will explain and generalize this result.

Allison Torsey, State University of New York at Buffalo
Analyzing the Dynamics of an Inflammatory Response to a Bacterial Infection in Rats

Sepsis is a serious health condition defined by an overactive immune response that causes severe damage to healthy tissue, often resulting in death. Mathematical modeling has emerged as a useful tool to investigate key elements of the immune response and thus offers a useful method for studying sepsis. Here, a system of four ordinary differential equations is developed to simulate the dynamics of bacteria, the pro-inflammatory immune response, anti-inflammatory immune response, and tissue damage. The model is used to assess the conditions under which health, aseptic (inflammation-driven) death, or septic (bacteria-driven) death is predicted in both the presence and absence of an induced E. coli bacterial infection in rats. Model parameters are fit to experimental data from rat sepsis studies. The model is used to predict the survivability range for an infection while varying the initial amount, growth rate, or virulence of the bacteria in the system. For highly virulent strains of bacteria, aseptic or septic death is predicted for very small levels of initial bacterial loads. Model predictions are also used to explain the experimentally observed variability in the mortality rates among rats.

Lee Trent, Rose-Hulman Institute of Technology
see **Kaleigh Roach**

Carlie Triplitt, University of Science and Arts of Oklahoma
see **Sarah Lubow**

Alexandra Tubbs, Denison University
see **Nicole Harris**

Sister Megan Therese Tunink, Benedictine College
Predicting Migration Timing of the Ruby-throated Hummingbird Using Multiple Regression

The Ruby-throated hummingbird (Archilochus colubris) is the only hummingbird species to migrate across a major geographical barrier in a single flight. They make this migration across the Gulf of Mexico two times a year to move between their wintering grounds in Central America and their breeding grounds in eastern North America. We used multiple regression to create a model in order to predict when the hummingbird species will first be sighted in the spring in any location in the eastern United States. Possible predictor variables in these analyses included weather conditions in both the sighting location and in a possible stopover location on the northern coast of the Gulf of Mexico, latitude of the sighting, and phase of the lunar cycle.

Maia Wichman, Grand Valley State University
Exploring The Earth Moon Problem for Doubly Outerplanar Graphs

The Four Color Theorem and its related research applied graph theory to the issue of map coloring. We will do the same, considering Ringel's Earth-Moon problem, where every country on Earth forms a single colony on the Moon. Now, we have two maps which can be interpreted as two planar graphs on the same vertex set; their union is called an Earth-Moon graph. The problem is then to determine the minimum number of colors needed to color the family of Earth-Moon graphs. In this talk, we discuss the history, motivation, and progress surrounding the Earth-Moon problem and its variations. More specifically, we consider tighter restrictions on the layers of Earth-Moon graphs. This research was conducted as part of the 2018 REU program at Grand Valley State University.

Fatima Zaidouni, University of Rochester
Threshold Optimization in Multiple Binary Classifiers for Extreme Rare Events

Classification on imbalanced datasets is a challenging problem where a high rate of correct detection is required in the minority class. We analyze the output of binary classification models used by Google, where the inputs are documents categorized as either predicted positive or negative against a certain threshold. In rare-event problems, positives have a prevalence of around 0.1% and it is expensive to estimate all documents. Therefore, the problem is reformulated using the correct labels [true positive (TP) or false positive (FP)] on a sample of the predicted positives, as determined by human raters.

It is important to pick an operating point (OP) on the TP/FP fitted curve whose position is adjusted to return the cost for one additional TP document in terms of the number of FP. We propose two solutions to select an optimal OP by maximizing the area under the curve (AUC): a graph-based and an analytic approach. The graph-based approach constructs a graph to select an optimal path in the threshold space that is then converted to a curve in the TP/FP space. The analytic approach estimates the AUC by minimizing a cost function. Our approaches improve over existing solutions by relating the TP/FP space to the threshold space and offer a business interpretation to the OP.

Camille Zaug, Seattle University
Frequency Downshift in the Ocean

Frequency downshift occurs when a measure of a wave's frequency (typically its spectral peak or spectral mean) decreases monotonically. Carter & Govan (2016) derived a viscous generalization of the Dysthe equation that successfully models frequency downshift in wave tank experiments for certain initial conditions. The classical paper by Snodgrass et al. (1966) shows evidence that narrow-banded swell traveling across the Pacific Ocean also display frequency downshift. In this work, we test the viscous Dysthe equation against the Dysthe equation, nonlinear Schrodinger equation, and the dissipative nonlinear Schrodinger equation to see which generalization best models the ocean data reported in Snodgrass et al. We do so by comparing the Fourier amplitudes, the change in the spectral peak and spectral mean, and conserved quantities representing mass and momentum between the ocean measurements and numerical simulations.

Hongyuan Zhang, Grinnell College
see **Alicia Ledesma Alonso**

Julie Zhang, University of Washington
Comparing Object Correlation Metrics for Effective Space Traffic Management

In the near future, the number of satellites in Low Earth Orbit (LEO) is expected to grow tenfold. Therefore it is important to determine optimal space traffic management systems under various conditions. One essential part of space traffic management is the problem of object correlation: Given an a priori distribution of each object in space at a given time and noisy measurements of unknown objects at a later time, how can one best associate each measurement to an object? This process of correlation depends on the choice of metric to quantify the likelihood that a certain measurement pairs with a certain object. Many metrics are already defined and explored in the literature, such as Mahalanobis, Bhattacharyya, Kullback-Leibler, and Optimal Control Distance. We contributed to this discussion a flexible simulation framework for comparing the performance of these metrics in a variety of scenarios while varying many simulation inputs. We focused our analysis on the cases where satellites move toward a pinch point or move away from a pinch point, as well as a case that simulates a more realistic LEO-type environment. The data shows a preference for Mahalanobis overall, although other metrics are superior in a few cases of interest.

Erin Zhao, Indiana University-Purdue University Indianapolis
Modeling Acute Blood Flow Responses to a Major Arterial Occlusion

Peripheral arterial disease (PAD) is a serious illness in which major arteries become blocked, causing reduced blood flow to peripheral tissues. Improved diagnoses and treatments for PAD require a more complete understanding of the changes that occur in vascular segments both proximal and distal to the site of occlusion; such information is investigated using mathematical modeling. Here, a mechanistic model of the vascular network of the rat hindlimb is developed to predict the immediate (acute) changes in vessel diameters and smooth muscle tone following femoral arterial occlusion. Vascular responses to changes in pressure, shear stress, and metabolic levels are modeled, and both resting and exercising conditions are simulated and compared with experimental data. Despite significant acute dilation of collateral arteries, model results show that the oxygen saturation and blood flow in the calf following occlusion are still reduced, and the deficits become more significant as activity levels increase. This suggests that long-term (chronic) structural adaptations are also necessary to restore blood flow to healthy, pre-occlusion levels.