The Twenty-Second Annual Nebraska Conference for Undergraduate Women in Mathematics

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TALK ABSTRACTS

PLENARY TALKS

Dr. Margaret (Midge) Cozzens
Distinguished Professor of Mathematics and Associate Director for Education at DIMACS
Rutgers University
You Can Do Anything You Set Your Mind To Do

You will learn what you need to know. You will seek out others to help you achieve. Your hard work will drive your success. Believe in yourself. These are words that have guided my life and career for over 60 years, words that my grandmother repeatedly told me while growing up. My life and career have not followed a straight path at any time, but throughout it all, there have been two driving passions – to learn something new and to educate others. I firmly believe that everyone can learn mathematics and learn how to use it to solve problems. I told my third grade teacher that I would teach mathematics when I grew up, and I did. I have taught high school through graduate school, including while I was a department chairperson at Northeastern University, Provost at the University of Colorado, President of the Colorado Institute of Technology, and today at Rutgers. I am an applied mathematician, which I define as starting with a problem and developing the mathematical tools to solve the problem. The problems come from many places and are influenced by the needs of the day. An REU student working with me three summers ago and I solved a problem posed in the 1970s and one which I had tackled in my thesis. This talk will describe this problem about food webs from ecology, as the path to its solution is, in itself, fascinating and it became very personal to me. I hope at the end of the talk you can see yourself in some of what I say, as well as opportunities for your own future.

Dr. Trachette L. Jackson
Professor of Mathematics
University of Michigan
Mathematical Biology and the Revolution of 21^{st} *Century Science*

Mathematical biology is an exciting interdisciplinary field that combines applied mathematics, scientific computing, biology, ecology, physiology, and medicine. This branch of mathematics is growing with phenomenal speed. For the mathematician, biology opens up new and exciting areas of study. At the same time, for the biologist, mathematical modeling offers powerful and complementary research tools that can provide insight into the complexity of biological systems. This talk will introduce the field of Mathematical Biology, highlight historical successes, and showcase ways mathematicians are currently tackling some of the most significant biological challenges our world is facing.

Talks by Undergraduate Students

Mandy Abernathy, Wisconsin Lutheran College
Theoretical Model of Blood Flow in the Human Retinal Vasculature

Glaucoma is a serious ocular disease that is characterized by damage to cells in the retina and results in irreversible vision loss. Most cases of glaucoma are attributed to fluid buildup that leads to high intraocular pressure, but clinical evidence shows that almost one-third of glaucoma patients do not exhibit elevated pressures in the eye. Other factors that may contribute to the progression of glaucoma are impaired blood flow and oxygenation of retinal tissue. Theoretical modeling provides a useful tool for predicting the impact of these hemodynamic factors on blood flow regulation and retinal oxygenation. In this study, a theoretical model of the human retina is adapted from a previously developed model of the mouse retina. Oximetry data from the human retina is used to convert the murine network to a human network. The model predicts oxygen distribution in the retinal arterioles using a Green's function approach to solve the diffusion equation for oxygen diffusion and calculates blood flow rate, pressure, and viscosity throughout the arterial network. Ultimately, this model provides an important approach for predicting retinal blood and tissue oxygenation within a realistic model of the arteries in the retina. A realistic model of blood flow in the retina makes it possible to further understand the role of impaired oxygenation in glaucoma and could improve diagnostic and treatment strategies for patients with glaucoma.

Lydia Ahlstrom, Keene State College
A Mathematical Investigation of Sol LeWitt's Wall Drawing 413

When one can find a mathematical concept in a work of art, it will enrich and illuminate both the mathematics and the art. Sol LeWitt was a 20th century artist whose work is closely associated with Minimalism and Conceptual art. His Wall Drawing 413, which can be viewed at the Massachusetts Museum of Contemporary Art in North Adams, MA, consists of 64 squares, each of which is divided into four smaller squares. Only four colors are used to fill the small squares. This talk will begin with an overview of Lewitt's art, followed by how Wall Drawing 413 can be interpreted in terms of permutations and groups.

Chelsea Alford, Furman University
Ella Morton, Furman University
An Integer Programming Approach to Assigning Faculty to Committees

Furman University faculty serve on numerous academic and administrative committees ranging from Study Away to Academic Discipline. The Nominating Committee is one such committee that is charged with assigning the faculty who will serve on each committee. Our research involved mathematically modeling the assignment problem with which the nominating committee is tasked, using an integer programing approach. The objective of the model was to maximize the faculty's preferences for assignment while satisfying various constraints, including meeting committee sizes, giving only one assignment per professor, and letting third-year committee chairs take a year off. Considerations regarding the balance of gender, faculty rank, and academic division also were incorporated into the model. We wrote a computer application in Microsoft Excel that can be used by the nominating committee for years to come without expert knowledge of the mathematical nature of the underlying assignment problem. The application is expected to save the Nominating Committee at least one hundred person-hours each year, and can easily adapt to changes in University policy.

Jess Appel, University of Kentucky
Katie Gravel, Massachusetts Institute of Technology
Annie Holden, Colby College
On Congruence Subgroups of the Braid Group

The braid group is a mathematical object that is invaluable to math and science since it records the movement of particles through time. We study one representation of the braid group, called the symplectic representation. The kernel of the mod N reduction of the symplectic representation is known as the level N congruence subgroup, denoted $B_n[N]$. Kordek-Margalit showed a nonconstructive quartic lower bound of the size of a generating set for the level four congruence subgroup. Our first theorem constructs a quintic generating set for the level four congruence subgroup. Stylianakis showed that for prime ℓ, $B_n[\ell]/B_n[2\ell] \cong \mathrm{S}_n$. Our second theorem shows this result can be generalized in different ways for odd and even ℓ.

Therese Azevedo, Sonoma State University
Investigating Math Anxiety within College Students

Math anxiety is common for many people, with female and minority students being particularly susceptible. Students may begin to develop a fear of math from many different types of experiences. A negative attitude toward math can reduce one's confidence and avoidance of the subject. Math anxiety can affect one's decisions in life and choice of career.

The researchers' goal in this study is to understand college students' perspectives and experiences with math to determine the causes and impacts of math anxiety. The study does this by surveying and interviewing current college students. Females and minority students are the researchers' main focus due to the lack of diversity within STEM. The findings the researchers present identify the possible causes of math anxiety. They hope these insights can inform the creation of treatments to mitigate the experiences that contribute to negative outcomes with math, especially for female and minority students.

Talia Blum, Massachusetts Institute of Technology
Unlikely Intersections and Portraits of Dynamical Semigroups

Classical algebraic dynamics studies the behaviors of single rational functions under iteration. Of particular interest are the possible phase portraits of preperiodic points; Baker proved that with finitely many exceptions every preperiodic portrait can be realized by a complex rational function. Following recent work by Hindes and Hyde-Zieve, we investigate portraits for several rational functions simultaneously acting on the same point set. In this case, we no longer expect these portraits to be realizable for arbitrary rational functions. This leads to several questions, such as which portraits with several rational functions have realizations, and what properties do the spaces of all such realizations possess?

We used a computer cluster to determine realization spaces for all portraits with several points and two polynomials of low degree. Surprisingly, many portraits had realization spaces with higher than expected dimension. We will present three main results: a sequence of portraits with positive-dimensional realization spaces for multiple rational functions acting on arbitrarily many points; a classification theorem for the realizable dimension of two-image portraits; and a realizable portrait with 28 quadratic polynomials acting on four points.

Kathryn Bolinger, Washburn University
Optimal Ranges for Cutoff Probabilities of Discrete Distributions Derived from Nonstandard Decks of Cards

By considering new arrangements of suits within a deck of cards we asked ourselves some probability questions. We created a variable arrangement of cards (a deck) within which there were a number of suits (S), a number of cards within each suit (N), and a number of cards not in any suit (U). We then asked a question: How many draws does it typically take for all the cards of one suit to be drawn? This lead to a more intricate question. What is the shortest interval within which 90% of the time one of the suits will be exhausted? Using the programming language R, we created a model with which we found some empirical answers. This turns out to involve finding the integral over the shortest interval that arrives at a particular value over a density function. In this talk we present some of our results in this exploration that used ideas from probability, calculus, statistics, and algorithms coded in R.

Meredith Bomers, Hope College
Developing a General Compton Scattering Cross-Section in Strong Magnetic Fields

Various X-ray space telescopes have detected steady soft X-ray emission originating from highly magnetized neutron stars known as magnetars. Within their magnetospheres, accelerated electrons interact with and boost X-ray photons through the quantum electrodynamic (QED) process, Compton scattering. This is preferred to produce the high-energy tails observed in magnetar thermal X-ray spectra. Through the implementation of Sokolov & Ternov (S&T) spin states, there exist analytic expressions for the spin-dependent lifetimes of excited-state particles in magnetospheres, required to determine the spin-dependent Compton scattering cross-section. We propose the development of correct, spin-dependent compact analytic expressions for the Compton cross-section to eventually be used in Monte Carlo simulations of magnetars' X-ray emission. With these expressions, we will graphically analyze the specific effects of initial and final spin states as well as the role of polarization in photon intensities. This will allow for more accurate and efficient Monte Carlo modeling of magnetars and may help in understanding distinct features between highly magnetized and conventional gamma-ray pulsars. This research is made possible by the generous support of a Clare Boothe Luce Research Scholars grant to Hope College from the Luce Foundation, the Michigan Space Grant Consortium, and the Hope College departments of Physics and Mathematics.

Andrea Boskovic, Amherst College
Online Algorithms for Allocating Products to Users

In order for an e-commerce platform to maximize its revenue, it must recommend customers items they are most likely to purchase. However, the company often has business constraints on these items, such as the number of each item in stock. In this problem, our goal is to recommend items to users as they arrive on a webpage sequentially in an online manner, in a way that maximizes reward for a company, but also satisfies budget constraints. We first tackle the simpler online problem in which the customers arrive as a stationary Poisson process and present an integrated algorithm that performs online optimization and online learning together. We then make the model more complicated but more realistic, treating the arrival processes as non-stationary Poisson processes. To deal with heterogeneous customer arrivals, we propose a time segmentation algorithm that converts a non-stationary problem into a series of stationary problems. Experiments conducted on large-scale synthetic data demonstrate the effectiveness and efficiency of our proposed approaches on solving constrained resource allocation problems.

Judith Brennan, Georgia Institute of Technology
Two-Stage Optimization Model for Fantasy Hockey Draft Picks

Fantasy hockey leagues consist of owners who draft teams of hockey players from the National Hockey League player roster. We determine the optimal team for fantasy hockey using linear integer programming. Each owner is assigned a draft position 1 through n and proceeds to draft a single player in a 1 to n then n to 1 pattern. Regression models determine which player statistics are statistically significant. These statistics act as a set of criteria for player weights. The 2000-2001 to 2017-2018 season statistics are used in determining these weights. The first stage of the program returns an optimal team composed of nine forwards, six defensemen, one utility player, two goalies and five benched players. The second stage then takes this team in as an input and determines the best player to draft. The simulation then runs the first stage again to update the optimal team, eliminating the players drafted by other owners. The second stage then determines the next best player to draft. This continues until a full team is drafted. Other owners draft from ESPN's top player list. The 2018-2019 season statistics are used to compare how the optimal team competes within the league and for that season

Shannon Bride, Colorado School of Mines
Jayde Thompson, Colorado School of Mines
A Mathematical Model of Surface-Mediated Enzyme Inhibition Under Flow

Damage to a blood vessel triggers a series of events within the vessel wall, commonly known as the coagulation cascade. This cascade begins when an integral membrane protein, tissue factor (TF), is exposed and interacts with proteins in the plasma. These interactions produce enzymes that amplify and stabilize the coagulation process. One of these stabilizing processes is called the tissue factor pathway inhibitor (TFPI), which regulates blood coagulation by inhibiting the enzyme production at the injury surface. Previous mathematical models of the coagulation cascade under flow have shown this stabilizing mechanism to be weak in comparison to the removal of enzymes by flow itself. However, these studies were limited to small injury sites, resulting in small TF patches. In this project, we used the finite element method to simulate advection-diffusion-reaction equations that represent the surface-mediated coagulation events. We then investigated how the enzyme production in the presence of TFPI is affected by various injury sizes of the exposed TF.

Rilee Burden, University of North Texas
McKenzie Fontenot, University of North Texas
Zero Forcing and Iteration Index for Gear and Helm Graphs

We will review the established helm graph and gear graph, H_n and Gr_n, and outline proofs for their zero forcing number, $Z(H_n)$ and $Z(Gr_n)$, and iteration index, $I(H_n)$ and $I(Gr_n)$, respectively. On this basis, we expand the structures of H_n and Gr_n, creating two new classes of graphs: generalized helm graphs, $H^*(m,n)$, and generalized gear graphs, $Gr^*(m,n)$. We will then outline the proofs for $Z(H^*(m,n))$ and $Z(Gr^*(m,n))$, as well as $I(H^*(m,n))$ and $I(Gr^*(m,n))$. We then discuss how the proofs for generalized helm graphs and generalized gear graphs relate to hub graphs, a new and larger class of graphs.

Emily Cairncross, Oberlin College
Stephanie Ford, Texas A&M University
Classifying Toric Surface Codes of Dimension 7

Toric codes are a class of error-correcting codes initially introduced by Hansen, where a code C is a k-dimensional subspace of $\mathbb{F}_q^n$, coming from a lattice polytope defining a toric variety. In particular, a toric surface code of dimension k is generated by some lattice convex polytope $P \subset \mathbb{R}^2$, where k is the number of lattice points in P. We analyze various codes by restricting our focus to polygons in $\mathbb{R}^2$. Previous authors, beginning with Little and Schwarz, have completed classifications of these toric surface codes with dimension up to $k = 6$. We classify toric surface codes with dimension $k = 7$, building on the methods of Soprunov and Soprunova as well as Luo, Yau, Zhang, and Zuo. We separate codes into groups based on minimum distance and further distinguish codes in the same group using finer invariants, such as the number of codewords of various Hamming weights. We also determine when unimodularly inequivalent polygons generate monomially equivalent codes. Moreover, we lay the groundwork for others to classify codes of higher dimension.

Maxine Calle, Reed College
The Tambara Functor Structure of the Trace Ideal

Given a group G, a Tambara functor $\underline{T}$ can be specified by an assignment of a commutative ring $\underline{T}(G/H)$ for each open subgroup $H \leq G$, along with the Tambara structure maps (transfer, norm, restriction, and conjugation). This research, joint with Sam Ginnett, leverages Tambara structure to study the trace homomorphism between the Burnside and Grothendieck-Witt rings. Considering a (profinite) Galois extension K/F with Galois group G, we can assemble the homomorphisms to obtain a Tambara functor morphism between the Burnside functor on G and the Grothendieck-Witt functor on K/F. The kernel of this map, known as the trace ideal, is a Tambara ideal of the domain. We determine the Tambara-theoretic generators of this ideal in the case where G is a (pro-)cyclic group. Applying our results, we calculate the trace ideal for extensions of finite fields.

Sierra Cartano, Francis Marion University
Classifying Stocks by Sector Using Persistent Homology

The goal of this research was to determine if we could identify the sector of any random stock from the energy, financial, or technology sectors with persistent homology. Homology is used for identifying holes in a shape, with persistent homology being used to identify holes that persist over time. We modeled the shape of a stocks data using the Vietoris-Rips complex. With 90 stocks as our training data, 30 from each sector, we classified 90 test stocks using persistent homology diagrams and the bottleneck distance. We then completed a statistical analysis of how well the model classified the test data.

Camille Carter, Brigham Young University
Reconstructing Historical Earthquakes with Markov Chain Monte Carlo

When performing seismic risk assessments for communities near a given fault zone, it is crucial to consider the characteristics of damaging earthquakes that have previously occurred in the region. However, precise seismic data has only been recorded in recent history, so for many damaging earthquakes, the only records about the event available to us are anecdotal historical records. We present a method for identifying likely characteristics of an earthquake given anecdotal information, including descriptions of shake intensity and resulting damages. Our framework uses Bayesian statistical inversion and Metropolis Hastings sampling to infer parameters including magnitude, location, and geometry of the fault plane. As an initial test case, we use our framework to reconstruct the great 1852 earthquake and tsunami of eastern Indonesia.

Fangu Chen, University of Michigan
Symbolic Powers of Defining Ideals of Veronese Rings

The dth Veronese ring is the ring of polynomials such that the degree of every term is a multiple of d. We can represent these Veronese rings as the quotient ring of another polynomial ring. We are interested in the defining ideal of this quotient ring. The nth symbolic power of a radical ideal I can be thought of as the set of polynomials that vanish up to order n along the solution set of I. Our research is concerned with the symbolic powers and the primary decomposition of the defining ideals of Veronese rings.

Jiayi Chen, Grinnell College
Sanah Suri, Grinnell College
Coloring of Algebraic Structures

Given a group G, the noncommuting graph $\Gamma(G)$ is the graph formed by taking the noncentral elements of G as the vertices, and making two vertices adjacent if they do not commute. We show that if G is a finite group, then the chromatic number of $\Gamma(G)$ is 3 exactly when $|Z(G)| = |G|/4$. We also determine the chromatic and clique numbers of $\Gamma(G)$ whenever $|Z(G)| = |G|/2p$ for some prime p. In addition, we discuss the properties of maximal and maximum cliques in $\Gamma(S_n)$, where S_5 ends up being a critical group toward understanding when $\Gamma(G)$ has the property that all maximal cliques are maximum.

Calli Clay, Saint Catherine University
Ella Graham, Saint Catherine University
Finding Structure in Texts with Topological Data Analysis

Text Analytics is a method for extracting machine-readable data from texts. Topological Data Analysis (TDA) is a mathematical approach for analyzing complex data. A common method from TDA is persistent homology, which approximates the shape of data by identifying n-dimensional holes in the data. Our goal was to investigate TDA's effectiveness as a Text Analytics tool by analyzing poetry forms, such as villanelles and sestinas, and music genres, such as rock and pop. Using R, we represent each line of text with a word count vector, then calculate a distance matrix composed of the pairwise distances between each point in the resulting vector space. The code uses packages to calculate persistent homology, create barcodes, and find pairwise bottleneck distances between barcodes. This process gives a method for detecting similarities and differences within forms and genres, laying the foundation for comparing categories of texts.

Parmida Davarmanesh, University of Michigan
Automating Artifact Detection in Video Games

The video game industry is the most influential form of entertainment in America and the world. Despite optimized and specialized gaming hardware and software, gameplay can be impaired with graphics errors, screen artifacts, and other forms of corruption that are labor intensive to detect. This research has explored methods to automate anomaly detection and classification. This was accomplished by machine learning models to classify each frame of a video game as glitched or normal. The primary challenge is the lack of labeled and catalogued gaming data. To circumvent this bottleneck, a database was generated by extracting images from gameplay videos and adding artifacts modeled after observed corruption to the images. This work also explored several ways to extract features from the images, such as Fourier spectra, the histogram of gradients, and graph Laplacians. Using the extracted features, multiple classifiers were built to detect different types of glitches. Finally, an ensemble model was constructed by combining the individual classifiers using logistic regression. The results were able to accurately predict real corrupted images with a high degree of accuracy, indicating this approach may be applicable to a wide range of corruption in visual image processing.

Erin Dawson, Colorado State University
Trees with 4 Edges Decomposition of Complete 3-Uniform Hypergraphs

A graph is a set of n nodes, coupled with a set of 2-element subsets of the nodes called edges. Hypergraphs are extensions of the concept of graph. For positive integers k and n with $2 \leq k \leq n-1$, a k-uniform hypergraph is a collection of k-element subsets of a set also called edges with n elements. The complete *k-uniform hypergraph of order n*, denoted by $K_n^{(k)}$, is the k-uniform hypergraph on n nodes with the property that every set of k nodes is joined by an edge. A *loose m-cycle* in $K_n^{(3)}$ is a graph with vertex set $\{v_1, v_2, \ldots, v_{2m}\}$ and edge set $\{\{v_1, v_2, v_3\}, \{v_3, v_4, v_5\}, \ldots, \{v_{2m-1}, v_{2m}, v_1\}\}$. A common problem in the study of graphs is the problem of deciding when a large graph or hypergraph can be partitioned (i.e., divided up or decomposed) into pieces that all have the exact same structure as some smaller hypergraph. In this project, we analyzed hypergraphs to see when they could be partitioned into smaller hypergraphs that had four edges.

Annabelle Eyler, Hood College
Motion Analysis to Quantitatively Assess the Level of Pain in Animals

Therapies for chronic pain are greatly desired and often tested in animals before used for people. However, animals do not tell us explicitly how they are feeling. Since movement is directly correlated to pain, the goal of this project was to develop an approach to quantitatively assess the level of pain in animals by analyzing the patterns and rhythms of their self-directed motion to gain insights into their level of engagement or arousal. To achieve this, a cutting-edge computer vision software called DeepLabCut was used. DeepLabCut is a deep convolutional neural network that requires user-defined annotations and training time to recognize specified key points on the animal. However, because this software requires user input, measurements can be inconsistent. We studied the average human error and took into account the shortcomings of DeepLabCut when training the software. After training the neural network, we processed the key point predictions to get rid of inaccurate and impossible data points. To process the raw output, we created an algorithm that uses the moving average and linear interpolation to correct the falsely marked keypoints. Once the datasets were corrected, we analyzed the motion of rats by computing the total distance moved, average velocity, acceleration, jerk, and fluidity of motion, as well as how long they interacted with new objects in their environment. This work was completed at the 2019 SURIEM REU at Michigan State University.

Asiya Falak, Arizona State University
Implementing Statistical Genetics to Evaluate the Forensic Identities among a Canine Population

A comprehensive population sample of genetic data obtained from canines is required to evaluate forensic identification metrics. With two forensic canine genotyping kits, we generated short tandem repeat (STR) profiles to establish a local database of canine allele frequencies. A Bayesian clustering analysis (STRUCTURE) confirmed the mixed breed statuses within the local canine sample and denoted a wide variation of alleles. By performing an analysis of molecular variance (AMOVA), we quantitatively showed that allele frequencies demonstrated high (88-89%) variation amongst individual dogs in the sample. Following the necessary linkage disequilibrium analyses on each generic marker prior to performing forensic probability calculations, we cross-referenced individual DNA profiles against our database to compute major identification statistics such as RMP, CPI, CPE, and CPD. Finally, we estimated inbreeding and heterozygosity to determine how effective the local dog sample was in comparison to nationwide data. The results of the analysis displayed comparable genetic diversity between the predominantly mixed-breed local sample and the national database with purebred samples. Overall, we have concluded that characterizing the rich variation in genotypes in local dog populations is necessary and useful to properly conduct canine forensic identification.

Grace Feterl, Augustana University
Nicole Kratz, Augustana University
Inspiring Youth in STEM through Math Modeling

Over the last decade, there has been a strong push to illustrate how teachers can engage students in the math modeling process. Knowing this, we created a math modeling competition that utilizes a variety of mathematical techniques. Specifically, our competition asks students to analyze the usage of renewable energy sources for a growing city. We will present the problem, corresponding lesson plans, implementation of a math modeling competition in various schools, and plans regarding the student assessment. We also will cover the mathematics of our chosen modeling problem, the various difficulties students will encounter, and possible solutions.

McKenzie Fontenot, University of North Texas
see **Rilee Burden**

Stephanie Ford, Texas A&M University
see **Emily Cairncross**

Ella Graham, Saint Catherine University
see **Calli Clay**

Katie Gravel, Massachusetts Institute of Technology
see **Jess Appel**

Amanda Gulley, Rockhurst University
A Game Theoretic Analysis of the RSU 9 School Board's Weighted Voting System

Local newspapers in Maine have brought to light a dispute that has been going on for the past few years involving the ten towns in the Mt. Blue Regional School District (RSU 9). This school district currently uses a weighted voting method that is supposed to adhere to the one-person, one-vote policy that comes from the 14th Amendment to the Constitution's Equal Protection clause. However, this has led to the two largest towns in the district having a majority of the directors and votes on the school board. This situation has left many in the smaller towns feeling as if their voices are not being heard or represented in school board matters. Some members from the smaller towns have tried to take certain actions to rectify this, such as demanding a legal opinion of the voting method in 2015 to starting a petition in 2018 to change the way voting power is distributed on this school board. In this presentation, we will discuss what weighted voting is, and we will also explain the Shapley-Shubik power index. In particular, we will use this above mentioned power index to analyze the per capita voting power of this school district, the voting power held by each member of the school board, and how bloc voting affects the distribution of voting power for these towns on the school board.

Hanna Hoffman, Harvey Mudd College
Pascal's Mystic Hexagon in Tropical Geometry

Pascal's mystic hexagon is a theorem from algebraic geometry – given some six points in the projective plane, and three points generated by extending opposite sides of the hexagon, the three points are collinear if and only if the six points lie on a nondegenerate conic. We instead investigate this theorem with a geometry instead over the tropical semiring, where addition and multiplication are (respectively) defined to be minimum and addition.

Annie Holden, Colby College
see **Jess Appel**

Amber Hu, Yale University
Fusing Visual and Textual Information to Determine Content Safety

In advertising, identifying web content safety is a significant concern since advertisers do not want brands to be associated with unsafe content. At the same time, publishers would like to open as much inventory as possible. Thus, a fine balance must be achieved to satisfy both advertisers and publishers. In this talk, we propose a multimodal machine learning framework that fuses visual and textual features from web pages to produce content safety predictions. The primary focus is on late fusion, a multimodal approach that involves combining final outputs of separate modalities, such as computer vision and natural language processing, to arrive at a decision. We developed a fully automated pipeline for late fusion, using support vector classifiers and random forest classifiers with parameter optimization, to perform multilabel classification of web pages into 10 threat categories. We also introduce work in early fusion, which involves extracting, fusing, and reducing the dimensionality of intermediate features from the separate models. Both of our late and early fusion methods obtain significant improvements over the algorithms currently in use in the industry.

Anita Koh, Pacific University
Volume Bounds for Trivalent Planar Graphs

Knot theory ties together topology and hyperbolic geometry. In particular, we study the volumes of hyperbolic planar trivalent graphs. A trivalent graph is a graph with three edges at each vertex. What makes these graphs hyperbolic is their trivalent property being translatable to thrice punctured spheres in the fully augmented link form. The connection between fully augmented links and trivalent planar graphs allows us to apply previous work on knots and links to a graph theoretic setting. Previous work in knot theory does not focus on the trivalent graph representation when looking at the volume of hyperbolic links. Exact volume bounds for graphs with up to 12 vertices are calculated. Purcell's sharp lower bound for the volumes of fully augmented links translates directly to a sharp lower bound for trivalent graphs. Moreover, a natural cell decomposition on trivalent graphs allows us to interpret both Agol-Thurston's tetrahedral and Adam's bipyramid upper bounds for link volumes in the graph theoretic setting. An infinite family of graphs, inspired by Agol-Thurston's infinite chain link fence, proves this upper bound is asymptotically sharp.

Elana Kozak, United States Naval Academy
Proving the Convergence of a Monte Carlo Tree Search to Brownian Motion

Monte Carlo Tree Search is a branch of stochastic modeling that utilizes decision trees for optimization. The method has been used in various types of artificial intelligence (AI), most notably for game play. This project imagines a "game" in which an AI player searches for an unknown stationary target. We analyze its selection policy, called the Upper Confidence Bound for Trees (UCT), and determine that its decisions resemble a random walk. Finally, we prove that as the bounds for our search grid go to infinity, the UCT selection policy for a Monte Carlo Tree Search converges to Brownian motion.

Nicole Kratz, Augustana University
see **Grace Feterl**

Cristina Lange, Brigham Young University
Identifying Gerrymandering Using MCMC

We present a mathematical analysis of Utah's 2012 political redistricting, using Markov-chain Monte Carlo methods to construct a large ensemble of alternative district plans that satisfy the legal requirements of contiguous districts with equal population. We compare the legislature's adopted plan in terms of many different measures for redistricting fairness, including partisan bias, mean-median score, efficiency gap, and the percentage of Republican vs Democratic voters in each district. We use precinct-level election results from the 2010 United States Senate election to estimate the distribution of voters' political parties, and we use the GerryChain library written by the Metric Geometry and Gerrymandering Group to construct the ensemble.

Lily Li, University of California, Berkeley
Classifying Homomorphisms from the Braid and Symmetric Groups

Given a large (potentially infinite) group, we wish to choose a subset of more manageable size that captures properties of the original group under homomorphisms. A candidate for such a representative subset is a totally symmetric set, developed by Chen, Kordek, and Margalit. Totally symmetric sets under homomorphisms are constrained under a strong condition: the image of a size-n totally symmetric set is either a totally symmetric set of size n or a singleton. We determined numerous classes of groups' totally symmetric sets and characterized homomorphisms from the symmetric group and braid group to large families of groups relevant to geometric group theory, such as free, solvable, dihedral, and Baumslag-Solitar groups. For those groups, we further drew conclusions on their behavior under homomorphisms by bounding the maximal sizes of totally symmetric sets and showing in many cases all possible homomorphisms of these groups factor through a cyclic group.

Sarah McCarty, University of Nebraska at Omaha
Sufficient Sampling to Characterize Generalized Cantor Sets

Generalized Cantor sets are the set of numbers between 0 and 1 whose digits when written in base N, also known as the scale factor, are restricted to a certain subset. Cantor sets can be created by infinitely and iteratively removing intervals. We show the intersection of two Cantor sets, with scale factors whose only common integer power is 1, is almost empty with respect to either Cantor set. Further, the Cumulative Distribution Function (CDF) of a Cantor set measures how much of the set is less than or equal to x. CDFs of Cantor sets are continuous and exhibit self-similarity. When restricted to the class of functions that are CDFs of a Cantor set, we show a bound on the scale factor is necessary in order to uniquely determine the CDF and corresponding Cantor set from a finite number of sample points. Finally, given a bound K on the scale factor, we show between $O(K)$ and $O(K^3)$ sample points of the CDF are needed to uniquely determine the CDF and the Cantor set. We use the self-similarity of the CDF, the almost-empty intersection of Cantor sets, and properties of Kronecker products to prove the bounds.

Addie McCurdy, University of St. Thomas
Exploring the Garden of Petaluma Knots

Random knots can be used to understand properties of knotting in natural systems, e.g. DNA. One model for random knots is the Petaluma model, whose configurations look like daisies when projected onto the xy-plane. For each permutation on $1, \cdots, 2n+1$, there is a corresponding Petaluma knot whose strand heights through the central crossing point are the sequential values of the permutation. We present our results in 1) determining relationships between the knot types of the Petaluma knots and their corresponding permutations, and 2) comparing the distribution of knot types in the Petaluma model to distributions of knot types in other random knot models and those seen in natural systems.

Ella Morton, Furman University
see **Chelsea Alford**

Sarah Naldo, United States Naval Academy
Electron-Phonon Coupling in Transparent Conductors with Sensitivity Analysis

Understanding the thermal properties of transparent conductors can give us insight to their capabilities in applications such as solar cells, lasers, video displays, and other optoelectronic devices. One thermal property that is relatively unexplored, especially in transparent conductors, is electron-phonon (ep) coupling. We are modeling ep coupling, the interaction of excited electrons with the surrounding lattice of atoms, in multilayer systems with transparent conductors. My research explores the sensitivity of the two-temperature model, which relates electronic heat carriers to vibrational heat carriers to individual parameters. The sensitivity analysis involves a custom least squares fitting technique and the investigation of the change in error of the model with respect to each parameter. A sensitive model, in which the error changes a lot as the parameter value changes, gives us more confidence in our experimental value. We compare this numerical sensitivity to an analytical solution in a simplified system in order to verify our understanding of the system. This work will enable experimental exploration of ep coupling to a vast set of materials beyond transparent conductors and beyond what has already been explored in literature.

Megan Oeltjenbruns, Wayne State College
Solving the Radiative Transfer Equation using the Radon Transform

Medical professionals around the world use CT scans to diagnose and treat various conditions. The mathematical tools that allow medical professionals to take and interpret CT scans are the Radon transform and its inverse transform, respectively. While both are indispensable in medical imaging, each transform has unique mathematical properties with interesting applications to partial differential equations, also known as PDEs. One such property is the ability of the Radon transform to intertwine any number of spatial derivatives in an equation. This allows us to reduce the dimensionality of certain types of PDEs. In this research, we are able to take a variant of the radiative transfer equation and solve it with spectrally accurate results. Our methods are computationally efficient and are easily applied to the radiative transfer equation.

Megan Osborne, University of Scranton
Probability Modeling of HIV Viral Blips

Understanding and observing HIV has been an ongoing goal for decades. The hindrance to curing HIV likely stems from the behavior of latent reservoirs in the body. The latent cells that make up these reservoirs are able to carry infection and can become actively infected. At some point, the viral load in the body may fall below the detection level, followed by potential random blips of detectable viral load. Since the virus is able to remain present in latent reservoirs during treatment, it is much more difficult to track and makes viral blips difficult to predict. In light of this information, we plan to carefully model and examine the dynamics of the cell interactions of the virus, especially the latent cells and virions. We plan on using stochastic and probability models to capture the dynamics of HIV given the unexpected behavior of these cell types. This type of modeling will give us an accurate depiction of the behavior of this randomness. In doing this, we propose to provide a practical method to interpret experimental data in the context of the HIV model, focusing on the random behavior of the infection. This research took place at the University of Michigan-Dearborn REU.

Connor Parrow, Hobart and William Smith Colleges
Pan Flutes and Other Graceful Labelings

In graph theory, a graceful labeling of a graph with n edges is the labeling of its vertices with some subset of the numbers 0 through n such that no label is re-used and if each edge is labeled with the absolute value of the difference of its incident vertices, then each edge is assigned a distinct label from 1 to n, inclusive. The Graceful Labeling Conjecture, which has been unresolved for more than 60 years, proposes that every tree can be gracefully labeled. In this talk we will discuss a method to gracefully label certain types of trees, Pan Flutes, and an application of the Delta Construction to get graciously labeled trees.

Madeline Preston, Furman University
The Happiest Hotel on Earth

The Walt Disney World (WDW) resort in Orlando, Florida, is a very popular travel destination for millions of guests annually. When planning a trip to WDW, guests have the option of 32 resort hotels and hundreds of off-resort hotels surrounding the WDW resort property. We developed a system to simplify the hotel selection process by creating a recommender system for the 32 resort hotels. Our system was developed using survey data from over 10,000 guests administered by touringplans.com. Our best model uses a multi-tiered approach of a decision tree followed by dot product-based scoring to match each user to a small group of similar hotels. This model takes into account how important multiple attributes are to a guest with respect to how much a particular hotel satisfied those given attributes. Our research exposed some shortcomings in the survey data and thus our model's results are not as strong as we many have desired. Future work includes developing an expert model that is more malleable to WDW updates thus keeping this recommender system relevant and helpful for guests.

Mikelle Rogers, Brigham Young University
A Comparison of Effective Resistance and Katz Distance for Certain Families of Graphs

Effective Resistance and Katz Distance are two graphs metrics used for link prediction. Link prediction is used in many areas such as social media, online shopping, maps and the spreading of disease. Effective resistance is calculated by treating the graph as an electric circuit and then determining the resistance between nodes. The Katz distance considers the number of paths of different lengths between two nodes, weights these lengths and then sums them. When used for link prediction, both algorithms provide an ordered list of likely links. In this presentation, we will consider which values of the Katz constant give the same ordering of missing links as the effective resistance. Specifically, we will consider the range of the Katz constant for paths, cycles, and linear two-trees (2-paths). The result of this work will allow mathematicians to know when to use which algorithm to predict links in a network based upon how the network's links are formed.

Alexa Shreeve, Davidson College
Extending the Systems Model of Platelet Homeostasis to Understand Platelet Dynmaics in ITP

Immune Thrombocytopenia Purpura (ITP) is an autoimmune disease that results in lower than normal platelet counts in a patient's circulatory system. The disease symptoms are a result of accelerated platelet destruction by autoantibodies against platelets. In a healthy individual, a feedback mechanism of the platelet production system controls an inverse relationship between platelet levels and Thrombopoietin (TPO), a glycoprotein and main stimulator of platelet production. This feedback mechanism, however, is inhibited in a patient with ITP. As a result, ITP is characteristic of lower than normal platelet counts, with relatively unchanged levels in TPO concentrations. To explore the mechanisms related to ITP, we extend a Pfizer Inc. ordinary differential equations model of platelet homeostasis in healthy individuals to include macrophage dynamics that result in accelerated platelet destruction. The new model is used to investigate changes in both total platelet levels and TPO concentrations affected by macrophage dynamics at varying degrees of ITP. We find that as we accelerate platelet destruction, TPO levels rise—a response seen in healthy individuals, but undesirable for modeling ITP. Therefore, to compensate for the rise in TPO levels we must change the parameters involved with TPO consumption to model the inhibited feedback loop that exists in ITP patients.

Brittany Sullivan, State University of New York at Geneseo
The Role of Eigenvalues of Threshold Graphs to Study Graph Invariants

A graph constructed by a recursive process of adding a dominating or isolated vertex to an initial single vertex graph is a threshold graph. Threshold graphs have applications in linear programming problems, resource allocation problems, and in psychology. In this project, we study the general properties of the eigenvalues of threshold graphs. In general, the eigenvalues/eigenvectors of a graph can be used to study graph invariants and can reveal structural properties in a graph. Using the Cauchy interlacing eigenvalue theorem and subgraph structure of a threshold graph, we prove a recently stated conjecture that no threshold graph contains an eigenvalue in the interval $\left[\frac{-1-\sqrt{2}}{2}, \frac{-1+\sqrt{2}}{2}\right]$ other than possibly the eigenvalues -1 and/or 0.

Sanah Suri, Grinnell College
see **Jiayi Chen**

Gabrielle Taylor, Clarkson University
Impacts on patient's IPR in relations to RA with Behavioral and Socioeconomic Factors

Many studies have shown that chronic diseases have large economic burdens on financial health of those afflicted. Conversely, it has been known that debt and financial pressures can increase levels of stress and blood pressure. Incidence of rheumatoid arthritis is strongly influenced by socioeconomic and behavioral factors, as well as health-care finance. Rheumatoid arthritis (RA) is a chronic autoimmune inflammatory disease that most commonly affects the joints. Using the National Health and Nutrition Examination Survey (NHANES) data, we develop a multiple regression model to explain participants' income-poverty-ratios (IPR) through associated factors' on reported rheumatoid arthritis diagnosis. Prior to conducting statistical analyses, appropriate preprocessing, and data cleaning were done through R Studio 34. We focus on the quantification of these associated factors' financial impact on rheumatoid arthritis participants by using ethnicity, age, and chronic stress as predictors. This study is expected to improve the knowledge-base and accurately assess how these factors impact the patient's financial health.

Katherine Thai, Rutgers University
Combining Genetic Algorithms and Machine Learning (CgALM) for Modeling Complex Systems

Essential for services such as communication, navigation, and weather prediction, satellite constellations must minimize loss of coverage while subject to constraints on space traffic and the number of satellites available. Genetic algorithms (GAs) offer a versatile method of optimization with demonstrated success in applied problems. In the case of a computationally expensive problem, the necessity of repeated fitness evaluations prevents convergence of a GA in a feasible time frame. Implementing a more efficient surrogate model to estimate the expensive objective function poses a potential solution to this dilemma. Recent advances in machine learning methods, in particular neural networks, make them a compelling candidate as a surrogate function. A GA incorporating an ensemble of neural networks as a surrogate function is evaluated on a set of canonical test problems, including those with discrete inputs and multimodal objective functions, and applied to the problem of constellation design. This modified GA performs 4.5 times faster than the basic GA. This work was done as part of Research in Industrial Projects for Students at the Institute for Pure and Applied Mathematics.

Jayde Thompson, Colorado School of Mines
see **Shannon Bride**

Nia Walton, Spelman College
Ruoyi Wang, Agnes Scott College
Bounds on Entropy in Subgroups of the Braid Group

The braid group is an algebraic structure that tracks the motion of "particles" with respect to time, as they moved around in the plane, not intersecting paths. Associated to each braid is a real number called its entropy, which encodes how complicated the braid is. In this work we characterize the minimal entropy of braids in certain subgroups of the braid group. Using constructions developed by Thurston and Penner, paired with results from mapping class group theory, we proved that the minimal entropy in the Braid Torelli group and all level m congruence subgroups of the braid group have constant upper and lower bounds, independent of the number of strands. We also showed this pattern of constant upper and lower bounds doesn't hold for all proper normal subgroups of the braid group, by constructing counterexamples from the commutator subgroup of the braid group. This is a joint work with Sidhanth Raman, supervised by Dan Margalit and Hyunshik Shin, and supported by the NSF through the Georgia Tech Mathematics REU.

Ruoyi Wang, Agnes Scott College
see **Nia Walton**

Mira Wattal, Johns Hopkins University
Commutators and the Magnus Representation of the Torelli Group

For a given surface, Σ_g^b, we can study its group of symmetries, its mapping class group – denoted by $\mathrm{MCG}(\Sigma_g^b)$. One natural representation of this group includes the symplectic representation, the action induced by $\mathrm{MCG}(\Sigma_g^b)$ on the first homology of Σ_g^b.

In the 1970s, Birman and Powell found a generating set for the kernel of this representation, called the Torelli group. While this discovery was an important development in the study of mapping class groups, the Torelli group is still not well understood.

This summer we hoped to demystify the Torelli group by studying its Magnus representation, an action of $\mathrm{MCG}(\Sigma_g^b)$ on the fundamental group of Σ_g^b. With this perspective, we found an infinite family of elements that live in the kernel of the symplectic representation, but not in the kernel of the Magnus representation. For $g \geq 3$ and $b = 1$, this family consists of commutators of Dehn twists about pairs of non-separating, simple closed curves with trivial algebraic intersection and non-trivial geometric intersection.

Elizabeth Whalen, University of St. Thomas
Knots on the Brain: Finding and Identifying Knots in Proteins

Recent studies have found knots in the backbones of some protein chains; for example 1%-2% of brain protein is the knotted UCH-L1 enzyme. It is believed that the knotting is related to the function of the protein, although the exact relationship is not known. Researchers believe that the location of the knots (the so-called "knotted cores") could provide critical information to understand the utility of the knot to the protein. However, finding these cores is not trivial. Proteins are open chains (i.e., they have two free ends), so traditional knot theory cannot be used to classify the knotting in such chains. While several definitions have been used to measure knotting in the protein chains, no single definition has been universally accepted. We propose a new definition for identifying the knotted cores of proteins and compare our computations to those in the KnotProt database.

Yixuan (Sherry) Wu, Georgetown University
Introducing a Flexible Multivariate Count Distribution that Accommodates Data Dispersion

Multivariate data analysis refers to the study of several random variables that are inherently correlated among themselves. While much of the field assumes a multivariate Gaussian/normal distribution for continuous data, there still exist open questions regarding how to properly model multivariate discrete/count data. The multivariate Poisson distribution is the most well-studied distribution for handling correlated discrete count data; however, the distribution is known to be constrained by underlying assumptions. In particular, the multivariate Poisson relies on its marginal distributions to be equi-dispersed (i.e., for its respective means and variances to equal). This work instead develops a flexible multivariate count distribution (motivated by the Conway-Maxwell-Poisson (CMP) distribution) that contains several multivariate classical distributions such as Bernoulli and geometric distributions as special cases. The talk will offer further background regarding this new research development, introducing the audience to the resulting distributional form and related statistical properties that stem from this multivariate CMP distribution.

Yufei Zhang, Saint Mary's College
Total Difference Labelings of Graphs

Inspired by graceful and total labeling of graphs, we introduce the idea of total difference labelings. A total difference labeling of a graph G can be obtained by labeling the vertices from a set of positive integers and then labeling the edges by taking the absolute difference of the end vertices, and then requiring that the labels form a total labeling of G. The minimum possible greatest label on G is called the total difference chromatic number. In this talk, we will determine the total difference chromatic number of paths, cycles, stars, and caterpillars.