The Twenty-Seventh Annual Nebraska Conference for Undergraduate Women in Mathematics

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TALK ABSTRACTS

PLENARY TALKS

Dr. Shelly Harvey
Rice University
TBA

TBA

Dr. Tasha Inniss
Spelman College
TBA

TBA

Dr. Ulrike Meier Yang
Lawrence Livermore National Laboratory
TBA

TBA

Talks by Undergraduate Students

Beppy Badgett, Furman University
Classifying Toroidal Cartesian Products of Graphs

Imagine a complete graph on five vertices, K_5. If we attempt to draw this graph on the plane such that no two edges intersect, we fail. However, we can draw K_5 on a torus such that no two edges intersect. In fact, K_6 and K_7 can also be drawn on a torus in this way. This naturally leads us to question: which graphs can you embed on the torus that do not embed in the plane? By taking the cartesian product of two or more graphs, we are able to construct more complex graphs with varying levels of repetition and symmetry that clearly reflect the structure of its components. This regularity makes classifying all planar cartesian products of graphs (graphs that can be embedded on the plane) relatively simple, however, when increasing the complexity of the surface to a torus, we observe a dramatic increase in the number of cartesian products of graphs that can be embedded on it. This talk outlines a partial classification of cartesian products of graphs that embed on the torus, highlighting future steps that can be taken to complete the list.

Ang Barrett, Northeastern University
A Bi-Lipschitz Co-orbit Embedding of Point Clouds

I will be covering an overview of research done by myself, Mitchell Wadas, and Dan Kiem during the Northeastern Math Summer Research Program this past summer. In this talk, I cover our exploration of clouds of n points in d-dimensional space, which can be expressed as $d \times n$ matrices. We consider these matrices up to permutation of columns. I will discuss our construction of an embedding of this orbit space under the permutation action of the n-symmetric group into m-dimensional Euclidean space in a way that preserves distances up to a small distortion. I will then discuss conditions under which this embedding is both injective and bi-Lipschitz. Lastly, I will cover our experimental derivation of specific parameters which yield small distortion.

Penelope Beall, University of Florida
Nancy Chen, Cornell University
Nava Minsky-Primus, Yale University
Investigating Constructions of Macaulay Poset and Rings

A poset is Macaulay if its partial order interacts well with some additional total order. Given two posets, there are several ways to combine them to obtain a new poset. For each poset operation, we can ask under what conditions do two Macaulay posets yield a poset that is still Macaulay. We identified classes of posets which remain Macaulay under certain operations. There are also algebraic analogs in the form of quotients of polynomial rings. This project was part of the 2024 Polymath Jr. program.

Madeline Blong, University of Wisconsin–Eau Claire
Community Detection of Indigenous Beaders on Instagram

Community Detection of Indigenous Beaders on Instagram involves roots in coding theory and graph theory. Given a data set, the objective is to detect underlying communities within a population. We view the given data set as the output of a noisy communication channel and use decoding techniques to reveal the underlying communities. Essentially, any population of a particular size has a set of allowed possibilities for clusters. Using techniques from coding theory, we hope to unveil a "best fit" community cluster formation from our gathered data. Through a sampling of 50 Indigenous beaders on Instagram, we gather data about Instagram follows to eventually decode to smaller community clusters. We imagine the community clusters could reveal tribal affiliation, stitching techniques, or geographic location.

Katelyn Buck, University of Texas at Austin
Multi-Dimensional Graphs Modeling Self-Assembling DNA Nanostructures

Employing tools from graph theory and linear algebra, we model the biological process of the creation of nanostructures from self-assembling DNA complexes. We represent k-armed branch junction molecules with tiles which are vertices in a graph with half-edges. The half-edges depict the cohesive-end types of a DNA strand. We aim to determine the minimum number of tiles and cohesive-end types necessary to form the complete complex of a given multi-dimensional graph structure. The problem of modeling DNA self-assembly is particularly challenging when considering graph families which change in multiple dimensions. In this research, we present the minimum number of tiles and cohesive-end types necessary to create the stacked book graphs, the square lattice graphs, and the Mongolian tent graphs, under different laboratory constraints. This research was completed at the Summer@ICERM REU Program at Brown University in 2023, and it is an ongoing project.

Tristan Bullock, Wake Forest University
Graphs, Diagrams, Knots, Oh My!

The Ising model uses quantum mechanics to capture phase changes or spin behaviors in lattices of molecules. The model receives a planar graph G as input whose edges are labeled with signs and maps it to a complex number. The same planar graph G also encodes—although not quite uniquely—a diagram D of a knot K. Interestingly, the complex number from the Ising model tells us something about the knot K. You will find out more in the talk. Expect lots of pictures to look at!

Nancy Chen, Cornell University
see **Penelope Beall**

Sophia Child, Colgate University
Emily Riley, Millersville University of Pennsylvania
Skew Zero Forcing Properties on Graphs

In this talk, we investigate the process of skew zero forcing (SZF) on a graph G. SZF is an iterative process initiated from a subset S of the vertices, wherein if a vertex has exactly one neighbor not in S, that neighbor is added to S. This process continues until S reaches a stable state. Our research primarily addresses two aspects of SZF: First, we explore conditions under which the SZF closure of the empty set coincides with the zero locus of the graph. We identify square grid graphs and a specific subset of generalized Petersen graphs as examples satisfying this condition. Second, we examine failed SZF sets, defined as subsets that do not lead to the inclusion of all vertices when the SZF rule is applied. We introduce the concept of the failed SZF number, which represents the maximum size of such sets within a graph. We compute the failed SZF number for hypercubes and Jahangir graphs, and we develop an algorithm to determine this number for trees. Additionally, we establish bounds for the failed SZF number in rectangular grids and generalized Petersen graphs. Our findings contribute to the understanding of SZF dynamics and its implications in graph theory.

Leona Coha, Augustana University
The Geodesics of the Universal Lipschitz Path Space of the Heisenberg Group

Homotopy is an important tool in studying topological spaces, in particular in the fields of metric topology, algebraic topology, and sub-Riemannian geometry. The homotopy classes of based Lipschitz paths in a based metric space are collected to define the universal Lipschitz path space, which under nice conditions is itself a based metric space. Moreover, for purely 2-unrectifiable metric spaces, in particular the Heisenberg group, the universal Lipschitz path space is a geodesic space. Thus, an explicit construction of these geodesics is desirable, which is what we find in this paper.

Catie Corchado, California State University, Fresno
Course Success and Peer Engagement: The Effects of Calculus Recitation Sessions at Fresno State

To combat a prevalent low success rate as college STEM majors take courses in the Calculus sequence, recitation support sessions focusing on group work and discussion were introduced at California State University, Fresno. This research aims to study the effects of this implementation in Precalculus, Calculus I, and Calculus II courses by investigating student perception and course success. Discussing the qualitative student-reported perceptions of recitation sessions and quantitative ordinal linear regression analysis of course outcomes will bridge the gap between how research views these sessions versus the impacted students' experiences. The results of this study highlight recitations' replicable assets and positive effects on course grades and student learning.

Mckayla Davis, Brigham Young University
Lydia Tolman, Brigham Young University
Mathematical Tools for Noninvasively Extracting Physiological Information from Pulsatile PPG and Spectrometer Signals

Pulsatile photoplethysmography (PPG) involves using light in various wavelengths to noninvasively measure volume changes in human capillaries or arteries due to heart beat. PPG signals have long been used to estimate blood oxygen levels (with a pulse oximeter), but looking at other wavelengths has the potential to help us identify other substances in the blood (like glucose) and also estimate properties of the cardiovascular system, like overall cardiac output. I will share some of the math that my collaborators and I have developed to try to explore this space.

Soleil Demick, Skidmore College
Modeling Growth of Invasive Watermilfoil in a Column

Watermilfoil (Myriophyllum) is an invasive genus of aquatic macrophyte to the Eastern United States and Canada. Its rapid growth forms dense monocultures which suppress local biodiversity and impede human activities such as boating and fishing. We adapt a prior aquatic macrophyte growth model to study watermilfoil in particular. Rather than modeling the biomass distribution itself, we use an advection-type PDE to model the distribution of plant tips within a column of water. Due to the uniformity of the plant along its length, the total columnar biomass may be inferred from the tip distribution. We perform numerical analysis of the model in varying regimes for parameters including water quality, column depth, light absorbance rate, and respiration rate, and we obtain analytical results concerning steady states. This study hopes to aid in the prediction and moderation of further milfoil development in the region.

Matilyn Douglas, Metropolitan State University of Denver
Violet Paukert, Metropolitan State University of Denver
Unlocking the Tactile World: Exploring Algebras with Braille

The set of 64 Braille characters, and particularly its subset of 26 Braille alphabet characters, is an interesting algebraic universe. By studying and creating valid algebras over the set, we will reveal underlying truths, which include structurally significant binary, unary, and nullary operations on the universe. Our results thus far are preliminary; however, all binary, unary, and nullary operations on the universe will be represented by 1-2 algorithmically defined binary operations which satisfy algebraic definitions of groups, rings, or fields. Upon identification of the Braille algebra, we will seek meaningful isomorphisms with other known algebras leading to a larger understanding of the Braille universe as a whole. The applications of such understanding are many and varied, a substantial example being enhanced computerized translation services. This process is an interesting examination of the interactions between operations and sets, as well as being representative of significant potential to improve accessibility.

Alyssa Ebeling, Wisconsin Lutheran College
Eliminating Eve's Eavesdropping: Using Algebra to Protect Your Data

Cryptography, the art of writing and solving codes, has been a major force acting in the shadows of history's greatest advances and conflicts. The influence of cryptography is not relegated to the past: the encryption and effective transmission of data are crucial in today's information age. As new methods in cryptanalysis are developed to attack ciphers, new ciphers are developed to withstand these attacks. For example, the Advanced Encryption Standard (AES), a cryptography algorithm used ubiquitously for the encryption of electronic data since 2001, has the multiplicative inverse function f_{inv} as one of its components. This function has a property called Almost Perfect Nonlinearity (APN), a property that increases AES's resistance to differential attack. Unfortunately, APNness does not provide resistance against integral attack, a more recent form of cryptanalysis. However, integral attack is prevented by a generalization of APNness called kth order sum-freedom. The function f_{inv} is kth order sum-free in some cases. Carlet's Conjecture considers the values of n for which f_{inv} from $\mathbb{F}_{2^n}$ to $\mathbb{F}_{2^n}$ has this key property of kth order sum-freedom. We proved select conditions of Carlet's Conjecture on kth order sum-freedom and found a surprising result when exploring the generalization of kth order sum-freedom of f_{inv} in $\mathbb{F}_{q^n}$, where q is an arbitrary prime number.

Sierra Edelstein, University of Florida
Uniform Distribution and Computability Theory: UD Randomness

We are working at the intersection of computability theory and analysis to study UD-random real numbers. We use computable functions: those that can be defined using a computer program that halts on every input. A sequence is uniformly distributed (UD) on the interval $[0, 1]$ if the frequency at which points appear in every subinterval of $[0, 1]$ is proportional to the length of the subinterval. In 2013, Avigad defined a UD-random real x as one for which the sequence $\langle [a_i \cdot x] \rangle$ is UD for every computable sequence $\langle a_i \rangle$ of distinct nonnegative integers. We study the randomness of n-computably enumerable (c.e.) subsets of $\mathbb{N}$: sets defined by allowing a computer to "change its mind" up to n times, on which natural numbers belong in our set. Formally, if we take a computable function $f : \mathbb{N} \times \mathbb{N} \to \{0, 1\}$, whose limit over s is the characteristic function of our set, we will find that the output $f(k, s)$ will change at most n times for any given k. We then use the characteristic function χ_A to represent A as the real number $.\chi_A(0)\chi_A(1)\chi_A(2)\ldots$. We prove that n-c.e. sets are not UD random for any n.

Nayda Farnsworth, Colgate University
Connections between Tridiagonal Matrices and the Fibonacci Sequence

The focus of this project is to better understand the connections between the class of $n \times n$ tridiagonal matrices and the Fibonacci sequence. Specifically, we consider the subclass of matrices denoted A_n in which all entries are zero except for the superdiagonal and subdiagonal entries, which are all one. Matrices in A_n arise as the adjacency matrices of the path graphs and hence are fundamental objects in the field of spectral graph theory, motivating our desire to better understand them and their properties. Interestingly, there seems to be a tantalizing connection between the characteristic polynomial $f_n(\lambda)$ associated with A_n and the $(n+1)$st Fibonacci number, denoted F_{n+1}. In their paper, Gullerud, Johnson, and Mbirika propose a conjecture stating that for all n, the roots of $f_n(\lambda) = F_{n+1}$ form an ellipse in the complex plane. At first glance, this seems accurate, but how can we *really* know? We must prove it. This project walks through the empirical and theoretical demonstration of the proposed conjecture, highlighting key strategies and outcomes for the curious mathematician.

Gwendolyn Flaherty, Agnes Scott College
A Construction of Set-Theoretic Solutions to the Yang-Baxter Equation

The Yang-Baxter Equation has applications in many fields of physics and mathematics, including statistical mechanics, quantum physics, and knot theory. In this talk, we focus on constructing set-theoretic solutions to the Yang-Baxter Equation on a set B of order p^k, where p is a prime number and k is a natural number. We generate these solutions using a certain class of matrices. We determine the size of this matrix class for any given dimension, and count the number of distinct set-theoretic solutions these matrices generate on the set B for small values of k. Explicit examples will be given.

Livia Fontana, Furman University
Isomorphic MCDO in Young's Lattice Intervals

A maximal chain descent order (MCDO) is a partial ordering on the maximal chains of a poset that is induced by a lexicographic shelling. We specifically examine the MCDO arising from a standard Young tableau (SYT), which induces a labeling (in particular, an EL-labeling) of an interval in Young's lattice. We discuss criteria for different SYT to induce isomorphic MCDOs. To this end, given a Young diagram with n boxes, we encode the filling using a $n \times n$ matrix of zeros and ones. Furthermore, we run an algorithm to determine whether the matrix corresponds to a standard filling of a Young diagram by identifying specific paths within the matrix. This tableau matrix and algorithm help us identify isomorphic MCDOs arising from a variety of different SYT.

Madeleine Forman, Keene State College
Using Neural Network Structure to Predict Network Behavior

Using simple network models called Combinatorial Threshold-Linear Networks (CTLNs), we simulate firing rates of neurons in the brain. The CTLN model is a system of ordinary differential equations whose solutions give the firing rate of each neuron. We specifically are trying to predict the firing sequences of small networks of 3-7 nodes based on their graphical structure. The goal of this research is to see if we can predict behavior from the structures of neural networks.

Aleksandra Gavrilova, Denison University
Mathematical Modeling of Pseudomonas Aeruginosa Quorum Sensing

Bacteria use quorum sensing to coordinate group behavior and regulate factors such as virulence and biofilm formation. Pseudomonas aeruginosa has multiple quorum sensing systems, including las and rhl. Systems can interact with each other in either a hierarchical or reciprocal manner, influencing signal production. Initially, it was believed that the las system solely influenced the rhl system; however, it was discovered that rhl also impacts las. Recent studies developed data-driven models based on ordinary differential equations to demonstrate the reciprocal characteristics of the rhl and las systems. We propose a reduced model and examine the biological and mathematical trade-offs of the simplification using dynamical analysis. We find that the two models have similar mathematical behavior in most regimes; however, the reduced model does not account for the behavior of the systems at low concentrations of bacteria. Additionally, the original model treats the population density dynamics of bacteria as constant, we compare this with two new models that incorporate variable population dynamics through simple logistic growth and cooperation. We find that the reduced model is biologically and mathematically accurate for high population growth rates and that for stronger cooperation there is no equilibria stability.

Bridget Ge, United States Military Academy
Modeling Degradation of Satellite-Based Capabilities Due to Nuclear Detonations in Space

As nuclear weapons continue to develop, this study aims to mathematically model the degradation of satellite-based capabilities resulting from a nuclear detonation in space. Focusing on the Global Positioning System (GPS), we used radiation exposure as a metric to predict the degradation of GPS constellations. The model assesses satellite operational impact, changes in overflight patterns, and their subsequent ground effects. We also developed a Python program to predict real-time satellite ground coverage upon nuclear detonation, which estimates and visualizes ground coverage. This research enhances the understanding of vulnerabilities in satellite-dependent systems and contributes to developing mitigation strategies for potential space-based nuclear incidents.

Alexandra Harley, Berry College
Predator-Prey Model for Analysis of Aedes Aegypti Population Dynamis with Holling Type II Consumption

In this presentation, we introduce a modification to a predator-prey model proposed for introducing a natural predator to the larval state of immature Aedes aegypti mosquitos, set in a local breeding basin in Cali, Columbia. The model describes the dynamics including age structure of the mosquito population, and building of the model formation in the previous paper, we introduce a Holling-type II consumption function for the natural predator. We prove existence and uniqueness of solutions, as well as give conditions for stability of equilibria mosquito for survival threshold. Numerical simulations are provided with introduction of an efficient predacious species in local mosquito breeding habitats.

Megan Hayduk, Hope College
Developing Intuitive Multiplication Fact Strategies with Young Students

The learning of basic facts, or sums and products of numbers 0-10 and their related differences and quotients, has always been a high priority for elementary school teachers. Current standards documents (e.g., the Common Core State Standards) expect multiplication fact mastery by the end of third grade, a daunting task. In an attempt to help mitigate that challenge, this project studied the impact of using visual imagery in the form of "Quick Looks" with dots in equal grouping patterns to encourage second graders to develop informal multiplicative understandings and strategies. The intervention consisted of six lessons taught by the researchers in second grade classrooms during May 2024. Comparison of pre- and post-interview results of 32 participating second grade students showed growth in all four aspects of procedural fluency (accuracy, efficiency, flexibility, and appropriate strategy use) for their multiplicative thinking skills.

Alaina Hogan, Grand Valley State University
Movement and Mathematics: Calculating Differences Between Dancers and Non-Dancers

In this talk, we study the physical, visual, and stylistic characteristics of trained and untrained dancing individuals. We determine these characteristics and prospective differences by acquiring data through motion capture equipment and conducting mathematical analysis. Participants in the study perform dance phrases in front of cameras while sensors record their movements in the XYZ coordinate plane. This coordinate data can then be analyzed; we can reveal patterns and information about the participants' movements using singular value decomposition and other data visualization techniques. We observe and calculate the differences that appear in movement between dancers and non-dancers.

Eliza Hogan, University of Michigan-Flint
Minimum Distances of an Infinite Class of Toric Surface Codes

In the field of coding theory, finding the minimum distance of a linear code is vital, as it determines a code's error correction capabilities. Finding this value is especially challenging for toric codes, a generalization of Reed-Solomon codes introduced by Hansen in 1997. Toric codes are k-dimensional subspaces of $\mathbb{F}_q^n$ obtained from toric varieties, which have a unique correspondence to integral convex polytopes in $\mathbb{R}^n$. Little and Schwarz (2007) outlined an elementary method using Vandermonde matrices to determine minimum distance formulas for toric codes given by simplices and boxes. To utilize this method for other polytopes, we first define a *staircase configuration*, a special collection of points in $(\mathbb{F}_q^*)^2$. We then prove that every Vandermonde matrix associated to a polytope from a certain class in $\mathbb{R}^2$ has non-zero determinant when evaluated at an appropriate staircase configuration. This class is described for any $\ell \in \mathbb{Z}^+$ by $\ell[0,(0,1)] + \ell[0,(1,0)] + \ell\Delta$ where $+$ denotes the Minkowski sum and Δ denotes the standard 2-simplex. Finally, we prove explicit minimum distance formulas for this infinite class of polytopes.

Shreya Jha, Georgia Institute of Technology
On the Use of Neural Networks for Accurate Transfer Function Approximation in Motor Control

Motors are widely used in various fields, ranging from the automobile industry to medical robots. In these fields, precise motor control is essential for optimal performance, especially in applications requiring high accuracy. Achieving this control requires an accurate transfer function, a mathematical representation of the behavior of a motor. While physics-based first principles can be used to compute this transfer function, this method becomes challenging and expensive with complex motors. As such, data driven methods are used instead, such as the Vector Fitting (VF) algorithm. However, VF struggles with poor predictions when using noisy data. To address this, we explore neural networks as an alternative approach for approximating the target transfer function. In particular, we investigate two types of neural networks: the ReLU neural network, and the rational neural network, which uses a rational activation function. Through numerical experiments, we find that neural networks outperform VF in noisy environments. Rational neural networks achieve similar accuracy to ReLU neural networks while requiring fewer parameters, and offer the additional advantage of enabling transfer function reconstruction. This combination of accuracy and interpretability makes rational neural networks a promising candidate for future motor control studies.

Serena Lewis, University of Arizona
Assessing the Differences between Clinical and Histologic Diagnoses of Chorioamnionitis

Chorioamnionitis (chorio) is a condition that can be diagnosed clinically or histologically, and discrepancies between these diagnoses may have important implications for maternal and neonatal outcomes. This study aims to assess the potential associations between clinical, demographic, and behavioral factors and the disagreement in chorio diagnoses. Using data of preterm births from a retinopathy of prematurity (ROP) registry, multinomial logistic regression models were developed to evaluate the relationships of select variables on a 4-level chorio diagnosis outcome. Key findings indicate that foul-smelling amniotic fluid and gestational age are significantly associated with both clinical-only and histological-only diagnoses of chorio, respectively. Additionally, Hispanic mothers and younger mothers were more likely to have a double positive diagnosis. At the same time, behavioral factors such as marijuana and tobacco smoking showed suggestive trends, though not statistically significant, that such use may increase the odds of clinical-only diagnoses. The small sample size in the clinical-only group may have limited the power to detect significant differences. Despite this, our findings highlight important cofactors that may influence discrepancies between clinical and histologic diagnoses. Results from this study can be useful to help contextualize the differences, allowing researchers to understand contributing factors better and improve future studies.

Raquel Liedtke, Drake University
Jenna Steffl, Drake University
Mitigating the Spread of Hospital Acquired Infection through Nursing Model and Hospital Layout: An Agent Based Modeling Study

Hospital-acquired infections (HAIs) pose significant risks to immunocompromised patients, particularly in inpatient settings where transmission via healthcare workers, such as nurses, is common. There are various nursing models that have been utilized as our healthcare system developed, such as the team model, functional or “specialist” model, and primary nursing model. Our work looked to build agent-based models to study the effectiveness of these various nursing models at preventing HAI transmission. Additionally, we investigated numerous hospital layouts to determine the optimal design for reducing infection spread through the hospital.

Neely Lovvorn, University of North Alabama
On the Number of Plane Graphs of Point Sets with a Small Convex Hull

A plane graph is defined to be an embedding on a planar point set such that the edges are non-crossing line segments. Such graphs are useful, since they model many real-world phenomena. For this reason, plane graphs have been extensively studied. In this project, we study properties of a random plane graph, uniformly chosen from the set of graphs that can be embedded over a fixed point set. While there is a significant body of knowledge about the behavior of random graphs, the standard tools break down for plane graphs. Hardly anything is known in this case. We derive bounds on the expected vertex degrees in a random plane graph of a fixed planar point set. Our proofs are based on cross-graph charging. That is, we move charge across vertices of different graphs, to obtain properties of an average graph.

Maisha Marzan, North Central College
Mathematical Model of a Multi-Host Pathogen System

Multi-host pathogens can create ecological links even between species interacting indirectly. This system encompasses multiple hosts, interconnected through a shared pathogen (Saprolegnia), with varying susceptibility to the pathogen which can have indirect negative effects on another host by amplifying the number of shed infective propagules. This interaction, like "apparent competition," can cause one host species to decrease in abundance disproportionately. We investigated 3 different types of alternative hosts, Vulnerable, Tolerant, and Saprobic hosts. Our goal was to understand how the presence of these different host species can influence infection in a target host (American bullfrogs). We developed distinct mathematical models for the populations of susceptible, hatched, dead, infected, and colonized hosts for each alternative host species, and incorporated a universal equation to represent the pathogen population. We used Python to develop numerical solutions and interactive plots for our model. We have calculated the community basic reproductive number, R0, and have conducted sensitivity analysis on our model parameters to determine the most important parameters affecting infection rates in our hosts. Our preliminary results conclude that alternative host species do have indirect negative effects on our target host abundance and that tolerant hosts negatively affect our target host the most.

Rebekah Mayne, University of San Francisco
There and Back Again: A Classification of Kaprekar 2-cycles

In 1949, D.R.Kaprekar observed a curious property of four digit integers arising from the following process: write a number's digits in both descending order and ascending order, and then find the difference between them. Iterating this process on any 4 digit number except for multiples of 1111 eventually produces the number 6174. When this process is applied to 6174, the result is again 6174. For numbers with more than 4 digits, the iterated process can either end in a constant number (as with 4 digits) called a Kaprekar constant, or a cycle of numbers, called a Kaprekar cycle. In 2011, S. Dolan found the structure of all Kaprekar constants. The purpose of our work is to find the structure of all Kaprekar cycles of length two, called 2-cycles. We found that there are infinitely many 2-cycles that separate into three distinct families.

Hannah Meit, Rhodes College
Two Partizan Games on Integer Partitions

We consider two variations of a game we call Column-Row Partizan (CRP), which is played between two players, Rob and Colleen, on a single integer partition λ or a multiset of partitions Λ. Colleen can remove any column, and Rob can remove any row. This leads to two possible games. In $CRPM$, if a Young diagram is broken in two by the removal of a row or column, the Young diagram is merged back together. In $CRPS$, any new diagrams created by such a removal are left separated with gameplay continuing on all remaining partitions. We provide a brief and complete solution to $CRPM$ and provide solutions and conjectures for $CRPS$ on certain shapes of partitions and multisets of partitions, including rows, columns, hooks, rectangles, and multisets of rows and columns. We hope to resolve current conjectures and obtain additional results for other shapes and multisets.

Nava Minsky-Primus, Yale University
see **Penelope Beall**

Grace Moberg, Colby College
Mathematically Modeling Disease Transmission in Long-Term Care Facilities

Clostridioides difficile, also known as *C. difficile*, is a prevalent cause of infectious diarrhea in healthcare facilities. Spread through the fecal-oral route and primarily through contact with spores on contaminated surfaces, *C. difficile* can cause severe diarrhea, stomach pain, and colitis. In particular, older populations have an increased risk for *C. difficile* colonization. Based on previous models of *C. difficile* transmission in hospitals, we developed a mathematical model of *C. difficile* transmission in long-term care facilities. We consider four classes of residents (susceptible, colonized, diseased, and quarantined) as well as three pathogen-carrying classes (high-traffic areas, low-traffic areas, and healthcare worker hands) in a compartment model to simultaneously capture the movement between classes and track the density of spores on these reservoirs, including how they contribute to disease spread. Using data from the Emerging Infections Program at the Centers for Disease Control and Prevention, we completed parameter estimations and further analyzed the impact of various parameters on incidence through sensitivity analyses. From these results, we identified mitigation strategies such as increased handwashing compliance, disinfection, and a higher ratio of healthcare workers to residents as being the most effective towards reducing incidence of *C. difficile*.

Violet Paukert, Metropolitan State University of Denver
see **Matilyn Douglas**

Jasmine Perez, California State University, Fresno
Impossible Corner Rotations on Variants of the Rubik's Cube

It is well-known in the Rubik's cube community that a classical $3 \times 3 \times 3$ cube cannot be fully solved with the exception of one corner rotated. This can be proved using invariants and modulo arithmetic. In some variants of Rubik's cubes, such as the Ivy Leaf, it is possible to rotate one corner and have the rest of the cube solved due its unique structure. However, in several other variants—for example, the Skewb—it is impossible to reach such a configuration. To prove this, we adapt the $3 \times 3 \times 3$ argument using group theory.

Sophia Pi, Northwestern University
Highlighting Limitations of Generative AI in Early Drug Discovery

In recent years, generative AI models have demonstrated promising advancements in the acceleration of drug discovery. Despite their growing popularity, there has been little work done to compare and highlight the limitations of existing models. In this project, we consider four "state-of-the-art" models - REINVENT4, CReM, SAFE-GPT, and COATI - and characterize the differences in performance and behavior across a variety of tasks. To do so, we develop a random forest pipeline for classifying molecules and producing meaningful low-dimensional visualizations, a comparative analysis of models' performance in a hit-to-lead optimization setting, and a large-scale case study of the distributions of molecules generated from each model. In particular, we develop a new sensitivity metric to capture the variance in these distributions.

Madhavi Prakash, University of California, Berkeley
Optimal Diffusion Kalman Filter for Lunar Time Synchronization

There has been an increasing eye towards developing a Lunar Time Estimation (LTE) system for use in a future Lunar Surface Communication Network (LSCN), with NASA, Aerospace and Nokia among a range of interested organizations. A reliable LSCN will require accurate, precise, and synchronized time estimates across a network of Lunar base stations. Bhamidipati et al. propose a modified Diffusion Kalman Filter for Lunar timing, which uses intermittently available Earth-GPS signals and measurements from neighboring stations to maintain accurate, synchronized time estimates. We first implement the Diffusion Kalman Filter proposed by Bhamidipati et al. Secondly, we explore optimizing our filter over the network weights used for diffusion to approach the error of a centralized filter estimate, which is optimal in the minimum mean squared error (MMSE) sense. We evaluate and compare the performance of a range of cost functions and optimization techniques, with applications to optimization on generalized diffusion algorithms. We also translate some results of the classical Diffusion Kalman Filter outlined by Cattivelli-Sayed to our LTE Diffusion Filter, most notably deriving the bias of our filter's estimate. Our effort more broadly aligns with renewed interest in Lunar missions and developing a sustainable human presence on the Moon.

Rachel Rainey, Keene State College
How Do US Mathematics Students Measure Up Against the Rest of the World?

The Trends in International Mathematics and Science Study (TIMSS) looks at achievement scores for 4th and 8th graders in 64 countries every four years to monitor global trends. The exam has 267 assessed math questions covering seven different topics. In initial data analysis of three topics areas, it was noticed the US tends to fall near the median of the country averages. However, it was found that when looking at individual questions the US scores widely vary from the median. This raised the questions: How did the US score on all the topic areas and do the above trends persist? How did the US perform on individual questions relative to the median throughout the topic areas? How do these scores reflect the US curriculum? To answer these questions, we will continue to analyze the TIMSS data and compare these results to the Common Core Math Standards for the US.

Emily Riley, Millersville University of Pennsylvania
see **Sophia Child**

Aimee Rohan Ramirez, United States Military Academy
Analysis of Factors Influencing a Marine's Heat Illness Symptoms

Exertional Heat Illness (EHI) is a critical occupational hazard affecting the operational readiness of U.S. Armed Forces, particularly in hot, humid environments. While past research has examined various intrinsic (e.g., gender, age, hydration, sleep) and extrinsic factors (e.g., environmental heat), few studies have evaluated their combined effects on EHI susceptibility in military personnel. This gap in understanding persists despite the importance of integrating both intrinsic and extrinsic factors to capture the complex etiology of EHI. To address this, we conducted a comprehensive study (2021-2023) on Parris Island, integrating intrinsic (e.g., gender, BMI, heat illness history) and extrinsic (e.g., WBGT index) variables. Our aims were to identify the impact of these factors on EHI risk, examine confounding variables, and use ordinal logistic regression to pinpoint key predictors. This analysis offers a clearer understanding of EHI risk, supporting better preventive strategies for military readiness.

Mia Schaefer, Colorado State University
Preservation of the Strong Spectral Property Under Edge Removal

The Strong Spectral Property of a matrix is a tool used in the Inverse Eigenvalue Problem. The class of graphs G for which every symmetric matrix associated with G has the Strong Spectral Property is denoted by GSSP. In this talk, we present families of graphs that belong to this class, especially families of dense graphs with few edges removed. In particular, we show that the complements of some trees, matchings, and certain unicyclic graphs are in GSSP. Moreover, we provide examples of families of graphs that do not belong to this class.

Sophia Schmidt, Drake University
Chord Progressions Modeled Through Abstract Algebra

This research explores how chord transitions create underlying algebraic structures. Building on Julian Hook's work on Uniform Triadic Transformations (UTTs), which categorize chord changes, we explore how these transformations can be viewed through the lens of group theory. While UTTs offer a way to analyze major and minor chord progressions, they fall short when it comes to more complex chords like seventh or inverted chords. Our goal is to expand UTTs to capture a broader range of chords and chord progressions, while using the properties of groups to study the musical significance of chord progressions.

Laura Seeberger, Murray State University
Understanding Perceptions of Power in a College Mathematics Classroom

Power is embedded in teacher and student actions within mathematics classrooms, whether that power is known or not. Incorporating equity and inclusiveness requires a deeper understanding of this power behind actions. In this study, we developed and administered a three part survey questioning college faculty and college students how they perceive power related to common actions within a college mathematics classroom. For example, participants were asked whether a college math teacher assigning homework increased, decreased, or did both to the teacher's power; then whether this action increased, decreased, or did both to a student's power. Analysis of 16 faculty and 121 students revealed faculty and students shared similar perceptions of power related to the role of the teacher and who has the power between a teacher and student. There was less agreement between faculty and students about the power associated with a student's actions.

Blessing Sithole, Keene State College
Mathematical Approaches to Securing Open-Source Syringe Infusion Pumps

Open-source medical devices, including syringe infusion pumps, have gained popularity due to their cost-effectiveness and adaptability. However, their open-source software introduces significant cybersecurity risks. This presentation addresses these vulnerabilities by integrating the distance sensor to enhance security and reliability through mathematical rigor. Using kinematics to model syringe movement, feedback control to minimize positioning errors, and error analysis with statistical validation to ensure sensor accuracy, this work demonstrates how mathematical methods can improve real-time monitoring, mitigate risks, and advance the safety of open-source medical devices.

Jenna Steffl, Drake University
see **Raquel Liedtke**

Emmerson Taylor, Park University
Structure Constants for the Universal Enveloping Algebras of Lie Algebras of Dimension Four and Smaller

Given a Lie algebra, L, over $\mathbb{R}$ or $\mathbb{C}$, the universal enveloping algebra of L, $\mathbf{U}(L)$, encompasses all noncommutative polynomials with coefficients from the field, where the variables are the basis elements of L. $\mathbf{U}(L)$ is also equipped with the commutator bracket, that is the relation $xy = yx + [x, y]$, which gives us a way to "straighten" or reorder products of basis elements. The Poincaré-Birkhoff-Witt (PBW) Theorem implies that every element of $\mathbf{U}(L)$ can be written uniquely as a linear combination of products of elements of L in any fixed order. The coefficients of the resulting linear combination are referred to as the structure constants. Previously Gourley and Kennedy derived a recursive formula for the structure constants of the universal enveloping algebra of $\mathfrak{sl}_2$. Then Chamberlin and Fernelius provided a closed formula for these constants. The goal of our research was to derive straightening identities in $\mathbf{U}(L)$ where L has dimension four and smaller, using Šnobl and Winternitz's classification of Lie Algebras of dimension six and smaller, as a way to represent the structure constants in $\mathbf{U}(L)$. We formulated and proved closed formulas for the structure constants of many of these algebras.

Anok Timothy, University of Nebraska-Lincoln
Alexander Polynomials of Symmetric Ribbon Knots

Ribbon knots are a special class of knots obtained by starting with $n + 1$ component unlinks or disks and resolving all attached bands. Every ribbon knot has a ribbon number, the number of ribbon intersections, or informally, the times a band crashes through itself. While it is possible for ribbon knots to have the same ribbon number and symmetric ribbon number, we find that this is not the case for all ribbon knots. Some might deem finding the symmetric ribbon number of a knot more difficult. A key indicator of symmetric ribbon numbers is Alexander polynomials. In this talk, we will determine the sets of Alexander polynomials of ribbon knots with symmetric ribbon numbers 2, 3, and 4. To do so, we restrict our attention to non-trivial, prime knots and involve presentation change methods.

Eliza Todd, Northern Arizona University
Iris Zepezauer, Northern Arizona University
New Results on Seymour and Sullivan Vertices in Directed Graphs

Given any vertex v in a digraph G, let $N^+(v)$ denote the set of vertices of G that are a path of length 1 directed away from v. Similarly, let $N^{++}(v)$ denote the set of vertices that are a path of length 2 directed away from v, and let $N^-(v)$ denote the set of vertices that are a length 1 path away from v but directed towards v. If $|N^{++}(v)| \geq |N^+(v)|$, the vertex v is called a Seymour vertex, and if $|N^{++}(v)| \geq |N^-(v)|$, the vertex v is called a Sullivan vertex. It is conjectured that every digraph has at least one Seymour vertex and at least one Sullivan vertex. We present new results on minimal lower bounds of numbers of Seymour and Sullivan.

Lydia Tolman, Brigham Young University
see **Mckayla Davis**

Ren Watson, University of Texas at Austin
Fixed Perimeter Analogues of Classical Partition Identities

In 2016, Straub proved that Euler's classic partition identity holds true for partitions with largest hook (perimeter) n. This inspired further study of the relationship between classical partitions and fixed perimeter partitions. We begin by proving and generalizing the fixed perimeter analogue of Andrews' S-T Theorem. We then extend the study of parity bias inequalities, first introduced by Kim, Kim, and Lovejoy in 2020, to the fixed perimeter setting and show using combinatorial methods that fixed perimeter analogues of many classical parity bias results can be proven and generalized. We finally prove Tribonacci recurrences for the number of fixed perimeter PED and POD partitions that is unique to the fixed perimeter setting.

Zoe Winston, United States Military Academy
A Metabolomics Analysis of Pregnancy Complications

Hypertensive disorders of pregnancy (HDP) pose a significant risk to maternal health. While previous research has linked HDP to inflammation and endothelial damage, comprehensive understanding of predictive biomarkers are lacking. This study aims to identify key metabolic disturbances associated with HDP by using a statistical battery to increase the robustness of predictive models. Using untargeted metabolomics data from first-trimester serum specimens from 51 HDP cases and 109 controls obtained from the Global Alliance to Prevent Prematurity and Stillbirth. We use a standard OPLS-DA and penalized regressions in order to provide a more robust analysis of which metabolites are significant in a particular disease. Stepwise logistic regression with Ridge regression demonstrated the highest predictive capability, reducing the original 3122 signals to 15 significant variables. Vitamin D3, bile acid synthesis, steroid hormone, and Vitamin E pathways were identified as significant in HDP, with pathway disturbances observed in gestational hypertension and pre-eclampsia cases.

Iris Ye, Carnegie Mellon University
A Helly's Theorem and Its Variations in Linear Partitions

Helly's theorem, along with its variations, has been extensively studied, with the fractional version being noteworthy. The fractional Helly's theorem of Katchalski and Liu says that in a finite family $\mathcal{F}$ of convex sets in $\mathbb{R}^d$, if at least some fraction of the total possible $(d+1)$-tuples in $\mathcal{F}$ intersect, then there must be at least some other fraction of all sets in $\mathcal{F}$ that intersect. Meanwhile, it has been proven that $d+1$ is the fractional Helly number for convex sets in $\mathbb{R}^d$. While this focuses on convex sets, our research studies the fractional version of Helly's theorem for linear partitions, which are defined as unions of subspaces that are in general position. Helly's theorem for linear partitions, proved by Arocha, Bracho, and Montejano, states that in a finite family $\mathcal{A}$ of linear partitions in $\mathbb{P}^n$, if every $\lfloor \frac{3(n+1)}{2} \rfloor$ or fewer linear partitions in $\mathcal{A}$ have a non-empty intersection, then all of them do. We upper bound the dual VC dimension of any finite family of linear partitions in $\mathbb{P}^n$ by $2^{n+1}-1$. Thus, by the theorem of fractional Helly for bounded VC-dimension of Matousek, we conclude that the fractional Helly number of any finite family of linear partitions in $\mathbb{P}^n$ is upper bounded by 2^{n+1}."

Nicole Zdunczyk, Drake University
Prospective Teachers Understanding of the Concept of Limits

This study examines secondary pre-service teachers' understanding of limits, which is a fundamental concept in calculus, with a goal of improving how limits are taught to future high school calculus students. It includes two components: interviews with secondary pre-service teachers and an analysis of high school calculus textbooks. The interviews evaluate teachers' conceptual grasp of limits and how they would respond to common student misconceptions. The textbook analysis focuses on how limits are presented, particularly the accessibility of the language used. By identifying gaps in both teacher knowledge and textbook explanations, this study aims to enhance instructional methods and better equip future teachers to teach limits effectively.

Iris Zepezauer, Northern Arizona University
see **Eliza Todd**