

**International Conference on
Geometric and Combinatorial Methods
in Group Theory and Semigroup Theory**

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ABSTRACTS

The linear nature of pseudowords

Jorge Almeida

CMUP/FCUP, Universidade do Porto

Relatively free profinite semigroups, whose elements are sometimes called *pseudowords*, have been recognized to play a crucial role in the theory of finite semigroups, namely in the Eilenberg/Schützenberger framework of pseudovarieties, which in turn is the suitable cadre for many applications in computer science. Yet, only for somewhat small pseudovarieties can one find in the literature structural descriptions of such profinite semigroups. For suitably large pseudovarieties, we construct a natural representation of pseudowords by certain labeled linear orders. In the case of the pseudovariety of all finite aperiodic semigroups, our representation is faithful.

This is joint work with Alfredo Costa, José Carlos Costa, and Marc Zeitoun.

On identities of indicator Burnside semigroups

Sergey Bakulin

Saint-Petersburg State University

Following Kublanovsky [1], any subvariety of a periodic variety generated by completely 0-simple semigroups is referred to as a Rees-Sushkevich variety. One of the important results concerning Rees-Sushkevich varieties, due Hall et al. [2], is that for each integer $n \geq 1$, the variety RS_n generated by all completely 0-simple semigroups over groups of exponent dividing n is finitely based. In [1] were constructed 13 finite semigroups and it was proven that a semigroup variety V is a Rees-Sushkevich variety if and only if it contains none of these semigroups. These semigroups are called indicator Burnside semigroup.

We provide a solution to the word problem and also provide a finite basis of identities for variety generated by indicator Burnside semigroups. We have proven that 10 of these semigroups generate small varieties and have described lattices of subvarieties. As a corollary we have shown that indicator Burnside semigroups generate hereditarily finitely based variety.

References

- [1] S. I. Kublanovsky, On the Rees-Sushkevich variety, unpublished manuscript.
- [2] T. E. Hall, S. I. Kublanovsky, S. Margolis, M. V. Sapir and P. G. Trotter, Algorithmic problems for finite groups and finite 0-simple semigroups, J. Pure Appl. Algebra 119 (1997), 75-96.
- [3] E. W. H. Lee, Combinatorial Rees-Sushkevich varieties that are Cross, finitely generated, or small, preprint.

Super-exponential 2–dimensional Dehn functions

Josh Barnard

University of South Alabama

We produce examples of groups of type \mathcal{F}_3 with 2–dimensional Dehn functions of the form $\exp^n(x)$ (a tower of exponentials of height n), where n is any natural number.

Commensurators of lattices in right-angled buildings

Angela Kubena Barnhill

Northwestern University

If G is a group and Γ is a subgroup of G , then the *commensurator* of Γ in G is the set of all elements $g \in G$ so that $g\Gamma g^{-1}$ is commensurable to Γ , i.e. so that Γ and $g\Gamma g^{-1}$ have a common finite index subgroup. In the Lie group setting, Margulis proved that a lattice is arithmetic if and only if its commensurator is dense. If G is the automorphism group of a locally finite polyhedral complex X , then *uniform lattices* in G are subgroups which act cocompactly on X with finite vertex stabilizers. If X is a tree, Liu showed that the commensurator of every uniform lattice is dense in G . When X is a right-angled building, we use a technique of “unfolding” to construct new lattices and use these lattices together with coverings of and actions on complexes of groups to show that the commensurator of the “standard uniform lattice” is dense in G . This is joint work with Anne Thomas.

Finitely presented solvable groups

Gilbert Baumslag

City College of CUNY

The recent focus on finitely generated solvable groups as geometric objects has underlined, in particular, how little is generally known about finitely presented solvable groups. The objective of my talk is to survey and discuss this class of groups and to highlight some of the intriguing open problems whose intractability have made progress so difficult.

Complexity of the Green relations of the Thompson-Higman monoids

Jean-Camille Birget
Rutgers University, Camden Campus

The groups $G_{k,1}$ of Richard Thompson and Graham Higman can be generalized in a natural way to monoids $M_{k,1}$ and to inverse monoids $Inv_{k,1}$. This is done by simply generalizing bijections to partial functions or partial injective functions. These monoids have remarkable properties, and they have close connections to circuits.

When inputs are words over a finite generating set of $M_{k,1}$, deciding the Green relations is in P. In order to exploit the connection of $M_{k,1}$ with circuits we use a generating set $\Gamma \cup \tau$ of $M_{k,1}$, where Γ is a finite generating set, and τ consists of the letter position transpositions on strings. Over $\Gamma \cup \tau$, deciding $\leq_{\mathcal{R}}$ is Π_2^P -complete, and deciding $\leq_{\mathcal{L}}$ is coNP-complete.

Over $\Gamma \cup \tau$, the characterization of the complexity of $\leq_{\mathcal{J}}$ and $\equiv_{\mathcal{D}}$ requires rather unusual complexity classes. Deciding $\leq_{\mathcal{J}}$ is coDP-complete, and deciding $\equiv_{\mathcal{D}}$ is $\oplus_{k-1} \bullet \text{NP}$ -complete.

The symmetric Torelli group

Tara E. Brendle
University of Glasgow / LSU

We will discuss work in progress with Dan Margalit related to a conjecture of Hain about the subgroup of the mapping class group consisting of elements which commute with a fixed hyperelliptic involution and which act trivially on homology.

Presentations galore

Colin M Campbell
University of St Andrews

At the time of the centenary of the Edinburgh Mathematical Society in 1983, two of the three honorary members were H S M Coxeter and W L Edge, both of whom, although better known as geometers, were also group theorists. More than a quarter of a century later, I realize that I have been influenced by these two mathematicians throughout most of my career. I will discuss group and semigroup presentations that have interested me over many years. These will include Fibonacci type presentations, presentations for simple groups, semigroup presentations for groups, the $F^{a,b,c}$ conjecture. Some questions have been answered but many open questions remain.

Automorphism groups of right-angled Artin groups
Ruth Charney
Brandeis University

Automorphism groups of free groups have many properties in common with linear groups. For example, they are residually finite, virtually torsion-free, have finite vcd, and satisfy the Tits alternative. Right-angled Artin groups interpolate between free groups and free abelian groups. We study which properties are shared by the automorphism groups of all right-angled Artin groups.

“Set-theoretical” solutions of the quantum
Yang-Baxter equation and a class of Garside groups
Fabienne Chouraqui
Technion

The quantum Yang-Baxter equation is an equation in the field of mathematical physics and it lies in the foundation of the theory of quantum groups. Let $R : V \otimes V \rightarrow V \otimes V$ be a linear operator, where V is a vector space. The quantum Yang-Baxter equation is the equality $R^{12}R^{13}R^{23} = R^{23}R^{13}R^{12}$ of linear transformations on $V \otimes V \otimes V$, where R^{ij} means R acting on the i -th and j -th components. “Set-theoretical” solutions of this equation are solutions for which V is a vector space spanned by a set X and R is the linear operator induced by a mapping $X \times X \rightarrow X \times X$. The study of these was suggested by Drinfeld. We establish a one-to-one correspondence between structure groups of non-degenerate, involutive and braided “set-theoretical” solutions of the quantum Yang-Baxter equation and Garside groups with a certain presentation. Moreover, we show that the solution is indecomposable if and only if its structure group is a Δ -pure Garside group.

The monomorphism problem in free groups
Laura Ciobanu
University of Fribourg, Switzerland

Let F be a free group of finite rank. We say that the monomorphism problem in free groups is decidable if for any two elements u and v in F , there is an algorithm that determines whether there exists a monomorphism of F that sends u to v . In this talk we will show that the monomorphism problem is decidable in F and we provide an effective algorithm that solves the problem. This is joint work with A. Ould Houcine.

Computing and approximating rotation distance

Sean Cleary

The City College of New York

Rotation distance measures the difference in tree shape between two binary trees of the same size. There are no known polynomial-time algorithms for computing rotation distance exactly. Computing rotation distance and finding optimal-length sequences of rotations are exactly measuring length and finding geodesics in Thompson's group F with respect to an infinite generating set. We give an algorithm for computing rotation distance exactly which is fixed-parameter tractable (FPT) in the parameter k , the rotation distance between the two trees. We give a linear-time estimation algorithm for rotation distance which is provably within a factor of $\sqrt{2}$.

This is joint work with Katherine St. John

Primitive elements in subgroups of free groups

Andrew Clifford

The College of New Jersey

Question F39 on the World of Groups list of open problems reads:

- (a) Is there an algorithm which, when given a finitely generated subgroup S of a free group F and an element g of F , decides whether or not there is an automorphism of F that takes g to an element of the subgroup S ?
- (b) The following special case of part (a) is especially attractive: given a finitely generated subgroup S of a free group F , find out whether or not S contains a primitive element of F .

In this talk, we provide an algorithm to answer part (b) of this question.

Morse theory and conjugacy classes of finite subgroups

Pallavi Dani

Emory University

Many groups have the property of having only finitely many conjugacy classes of finite subgroups. Examples include hyperbolic groups, $CAT(0)$ groups, mapping class groups, and automorphism groups of free groups. Do subgroups of these groups inherit this property? The answer turns out to be no. I will describe how combinatorial Morse theory can be used to produce counterexamples. This is joint work with Noel Brady and Matt Clay.

Presentations of automorphism groups of right-angled Artin groups

Matthew B. Day

California Institute of Technology

We find explicit finite presentations for the automorphism groups of all right-angled Artin groups. These presentations generalize the presentations given by McCool for automorphism groups of free groups. As an intermediate step, we generalize the peak-reduction theorem for free groups (Whitehead's theorem) to right-angled Artin groups.

The subword reversing method

Patrick Dehornoy

Université de Caen, Fr

Subword reversing is an algorithmic method for constructing van Kampen diagrams by referring to a preferred direction. Although reversing cannot work for every (semi)group presentation, it proves to be relevant in many nontrivial cases. Analyzing one example in detail, we shall summarize the main known results about the range of the method, its uses, and its efficiency.

Computing kernels of finite monoids

Manuel Delgado

Centro de Matemática da Universidade do Porto

The Rhodes type II conjecture, which proposed an algorithm to compute the type II subsemigroup of a finite monoid, has been first proved by Ash and independently by Ribes and Zalesskii. Both proofs are deep and led to interesting developments. The type II subsemigroup of a monoid is usually named *kernel* of the monoid, relative to the pseudovariety of all finite groups. The translation of the referred conjecture given by Pin and Reutenauer into a problem on profinite groups is at the base of Ribes and Zalesskii's solution and inspired the solution of analogous problems: computing kernels relative to other pseudovarieties of groups. Among these are pseudovarieties of p -groups (Ribes and Zalesskii), the pseudovariety of all abelian groups (Delgado) and other pseudovarieties of abelian groups (Steinberg). In the talk I intend to make a survey of known results on relative kernel computations and emphasize how effective computations can inspire theoretical results, namely in what concerns iterations of the relative kernel operator.

Generators and relations for singular semigroups of transformations

James East

University of Sydney

Every semigroup embeds in some transformation semigroup (Cayley's Theorem) and every semigroup is a homomorphic image of a free semigroup (every semigroup has a presentation by generators and relations); in the intersection of these two theories is a large and growing literature on presentations of transformation semigroups. When a semigroup S does not have an identity element, the Cayley representation maps the elements of S to *non-surjective* transformations. For finite sets, non-surjective is equivalent to non-*injective*, and the semigroup $T_n \setminus S_n$ of all non-invertible transformations arises—the so-called *singular subsemigroup* of T_n . In this talk I will discuss presentations for $T_n \setminus S_n$, as well as the singular subsemigroups of other related semigroups, including the partial transformation semigroup and the symmetric inverse semigroup. If time permits, I will show how these methods apply to the much larger partition semigroups, and discuss some unexpected results on idempotent generation.

A Short Proof that a CAT(0) cubical complex is a space with walls

Daniel Farley

Miami University (Ohio)

I will present a short proof that any hyperplane in a CAT(0) cubical complex X separates X into two convex complementary components. Sageev's original proof of this fact used a system of Reidemeister moves. The proof in this talk shows that his result is a corollary of Gromov's link condition. I will also show how to establish some of the combinatorial properties of CAT(0) cubical complexes without the use of the Reidemeister moves.

On the surface group conjecture

Ben Fine

Fairfield University

Divided-difference operators and diagrams

D. G. FitzGerald

University of Tasmania

The divided-difference operators, associated with Newton, are used in updating polynomial interpolations of bivariate data. They are nilpotent and generate a J -trivial semigroup. A presentation is known, having normal forms reminiscent of the symmetric group. This leads to a diagrammatic representation and corresponding moves of Reidemeister type. The interactions of the generators with transpositions from the symmetric group suggest that various larger semigroups are worthy of study too. Corresponding presentations and representations will be discussed.

Higher-dimensional Dehn functions of some abelian-by-cyclic groups

Max Forester

University of Oklahoma

I will discuss the determination of higher-dimensional Dehn functions for certain abelian-by-cyclic groups. These examples serve to show that the set of k -dimensional isoperimetric exponents of finitely presented groups is dense in the interval $[1, \infty)$ for $k \geq 2$. In particular, there is no analogue of Gromov's gap $(1, 2)$ in the isoperimetric spectrum when considering higher-dimensional isoperimetric functions.

Hard Problems in Group Theory

Bob Gilman

Stevens Institute of Technology

It is well known that difficult or even unsolvable computational problems can be easy most of the time, i.e., for most instances. Many problems from group theory are of this type. On the other hand there do not seem to be any examples known of concrete problems which are provably hard on a positive fraction of all instances. Group theory appears to be a good place to look for such problems; we will discuss progress which has been made in this direction.

Bisimple monogenic orthodox semigroups and their lattice isomorphisms

Simon M. Goberstein

Stanford University and California State University, Chico

We give a complete classification and description of the structure of all bisimple orthodox semigroups generated by a pair of mutually inverse elements and show that “almost all” semigroups of that class are determined by the lattices of their subsemigroups in the class of all semigroups.

Restriction semigroups and inductive constellations

Victoria Gould

University of York

The Ehresmann-Schein-Nambooripad (ESN) Theorem, stating that the category of inverse semigroups and morphisms is isomorphic to the category of inductive groupoids and inductive functors, is a powerful tool in the study of inverse semigroups. Armstrong and Lawson have successively extended the ESN Theorem to the classes of ample, weakly ample and weakly E -ample semigroups. A semigroup in any of these classes must contain a semilattice of idempotents, but need not be regular. It is significant here that these classes are each defined by a set of conditions *and their left-right duals*.

Recently, a class of semigroups has come to the fore that is a one-sided version of the class of weakly E -ample semigroups. These semigroups appear in the literature under a number of names: in category theory they are known as *restriction semigroups*, the terminology we use here. In a restriction semigroup, there is a well defined notion of domain, but *not* of image. In a recent article with Christopher Hollings, we show that the category of restriction semigroups, together with appropriate morphisms, is isomorphic to a category of partial semigroups we dub *inductive constellations*, together with the appropriate notion of ordered map, which we call *inductive radiant*.

Homotopical and homological finiteness conditions of monoids and their subgroups

Robert Gray

University of St Andrews

A common approach in semigroup theory is to try and reduce a given semigroup theoretic problem to a problem about groups. For example, one may try to reduce the problem of understanding a semigroup to that of understanding its maximal subgroups. In this talk I will discuss how the finiteness conditions satisfied by a semigroup influence, and conversely depend on, the finiteness conditions satisfied by its maximal subgroups. Specifically, I will present results about the homotopical finiteness condition finite derivation type (introduced by Squier (1994)), the homological finiteness condition FP_n and, relating to these, the property of being presented by a finite complete rewriting system. The results presented include joint work with A. Malheiro and joint work with S. J. Pride.

Ergodic properties of boundary actions and Nielsen–Schreier theory

R. Grigorchuk

Texas A&M University

Presented results are obtained in collaboration with V.Kaimanovich and T.Smirnova-Nagnibeda.

We study the basic ergodic properties (ergodicity and conservativity) of the action of a subgroup H of a free group F on the boundary ∂F with respect to the uniform measure. Our approach is geometrical and combinatorial, and it is based on choosing a system of Nielsen–Schreier generators in H associated with a geodesic spanning tree in the Schreier graph $X = H/F$. We give several (mod 0) equivalent descriptions of the Hopf decomposition of the boundary into the conservative and the dissipative parts. Further we relate conservativity and dissipativity of the action with the growth of the Schreier graph X and of the subgroup H (cogrowth of X), respectively. On the other hand, our approach sheds a new light on entirely algebraic properties of subgroups of a free group. We also construct numerous examples illustrating the connections between various relevant notions.

Artin Groups of Large Type are Automatic

Derek Holt

University of Warwick

An *Artin Group* is a group defined by a presentation

$$\langle a_1, \dots, a_n \mid (a_i, a_j)_{m_{ij}} = (a_j, a_i)_{m_{ij}} \ (0 \leq i < j \leq n) \rangle$$

where $(x, y)_k$ denotes an alternating product $xyx \cdots$ of length k beginning with x , and each m_{ij} is an integer with $m_{ij} \geq 2$. We also allow $m_{ij} = \infty$, in which case the corresponding group relation is absent. The Artin group is said to be of *large type* if each $m_{ij} \geq 3$.

It is conjectured that all Artin groups are biautomatic. This has been proved by Charney for Artin groups of finite type (corresponding Coxeter group finite), by Peiffer for those of extra-large type (all $m_{ij} \geq 4$), and by Brady and McCammond for Artin groups of large type with at most three generators. For general Artin groups the conjecture remains open.

In this talk, I shall briefly discuss my recent proof with Sarah Rees that all Artin groups of large type are (shortlex) automatic, and that their geodesics form a regular set. The proof is purely combinatorial. We hope to be able to address the question of their biautomaticity in the near future.

Semigroup identities in the monoid of two-by-two tropical matrices

Zur Izhakian

Bar-Ilan University

We show that the monoid $M_2(\mathbb{T})$ of 2×2 tropical matrices is a regular semigroup satisfying the semigroup identity

$$A^2 B^4 A^2 A^2 B^2 A^2 B^4 A^2 = A^2 B^4 A^2 B^2 A^2 A^2 B^4 A^2 .$$

Studying reduced identities for subsemigroups of $M_2(\mathbb{T})$, and introducing a faithful semigroup representation for the bicyclic monoid by 2×2 tropical matrices, we reprove Adjan's identity for the bicyclic monoid in a much simpler way. (Joint work with Stuart W. Margolis)

Solution of the membership problem for certain subsemigroups in one-relator groups with a small cancellation condition

Arye Juhász

Technion - Israel Institute of Technology

Let F be a free group freely generated by $X := \{x_1, \dots, x_n\}$, $n \geq 2$, let R be a cyclically reduced non-empty word in F and let G be the one-relator group presented by $\mathcal{P} = \langle X | R \rangle$. Let $X^{-1} = \{x_1^{-1}, \dots, x_n^{-1}\}$ and for convenience define $x_{n+1} = x_1^{-1}, \dots, x_{2n} = x_n^{-1}$. Thus, x_1, \dots, x_{2n} are semigroup generators of G . For a reduced non-empty word W in F given by $W = x_{i_1}^{\alpha_1} \cdot \dots \cdot x_{i_m}^{\alpha_m}$, $\alpha_i \geq 1$, let $\text{Supp}(W) = \{i_1, \dots, i_m\}$, $1 \leq i_j \leq 2n$ and for a non-empty set T of reduced words let $\text{Supp}(T) = \bigcup \text{Supp}(W)$, $W \in T$.

Very little is known on the solvability of the membership problem of subgroups and in particular of subsemigroups of one-relator groups. The membership problem is not known to be solvable for subgroups or subsemigroups even for hyperbolic one-relator group.

In this talk we consider subsemigroups of G generated by proper subsets of $\text{Supp}(R)$ (or $\text{Supp}(R^{-1})$) and some related subsemigroups. Our first result is the following.

Theorem 1 *Let notation be as above and let R be the symmetric closure of R in F . Suppose that R satisfies the small cancellation condition $C'(\frac{1}{4}) \& T(4)$. Let H be a subsemigroup of G with full preimage \tilde{H} in F . Suppose that R has no cyclic conjugate $UVU^{-1}W$, reduced as written with some combinatorial restrictions on U, V and W . If $\text{Supp}(\tilde{H}) \not\supseteq \text{Supp}(R)$ and $\text{Supp}(\tilde{H}) \not\supseteq \text{Supp}(R^{-1})$ then H has solvable membership problem.*

The growth rate of an endomorphism of a group

Delaram Kahrobaei

City University of New York

In this talk, I will discuss the notion of growth rate of a group endomorphism; introduced in 1978 by Bowen. I also give alternative definitions in the sense of Falconer and Bridson. I show how to calculate the growth rate of an endomorphism of a few classes of groups, particularly abelian, nilpotent and polycyclic groups; as well as semidirect products of groups. I will discuss some natural questions arising from this work.

This is a joint work with Kenneth Falconer from University of St Andrews and Benjamin Fine from Fairfield University.

Small overlap monoids

Mark Kambites

School of Mathematics, University of Manchester

Small overlap conditions are natural combinatorial conditions on semigroup and monoid presentations, which serve to limit the complexity of derivation sequences between equivalent words. Introduced and first studied by J. H. Remmers, they are the natural semigroup-theoretic analogues of the small cancellation conditions extensively used in combinatorial and geometric group theory. We present a number of results concerning presentations satisfying the small overlap condition $C(4)$. These include the fact that any semigroup or monoid with such a presentation admits a linear time solution to the word problem, a regular language of linear-time computable normal forms, an analogue of Kleene's Theorem, and a decision algorithm for the rational subset membership problem.

Properties of fully residually free (limit) groups

Olga Kharlampovich

McGill University

Finitely generated fully residually free groups (limit Groups) play a crucial role in the theory of equations and first-order formulas over a free group. It is remarkable that these groups, which have been widely studied before, turn out to be the basic objects in newly developing areas of algebraic geometry and model theory of free groups. Recall that a group G is called fully residually free if for any finitely many non-trivial elements in G there exists a homomorphism of G into a free group, such that the images of all these elements are non-trivial. I will give a survey of the present state on knowledge of these groups.

Surface subgroups of doubles of free groups

Sang-hyun Kim

the University of Texas at Austin

(Joint work with Henry Wilton) A long standing conjecture by Gromov asserts that a one-ended word-hyperbolic group contains a subgroup isomorphic to the fundamental group of a closed hyperbolic surface. An infinite family of hyperbolic groups can be obtained by taking doubles of free groups amalgamated along root-free words. Using group cohomology and 3-manifold theory, Gordon and Wilton recently provided certain sufficient conditions for these groups to contain surface subgroups. By realizing a double as the fundamental group of a non-positively curved square complex, we provide several other sufficient conditions for the double to contain a surface group; in particular, we answer a question (on finding a surface subgroup in a specific double) raised in the paper by Gordon and Wilton. These sufficient conditions on the amalgamating words are combinatorially given. The results presented here will not rely on the paper by Gordon and Wilton.

Biautomaticity and $CAT(0)$ simplicial 3-complexes

Rena Levitt

Saint Mary's College of California

In this talk, I will discuss the following result: groups acting geometrically on $CAT(0)$, simplicial 3-complexes are biautomatic. As a consequence of the proof, $CAT(0)$ simplicial 3-complexes are in fact simplicially nonpositively curved. I will then discuss current attempts to generalize this proof to $CAT(0)$ simplicial complexes of arbitrary dimension by focusing on specific subcomplexes, namely envelopes of geodesics.

Submonoid membership in metabelian groups

Markus Lohrey

University of Leipzig (Germany), Institut für Informatik

In the talk, we will show that membership in finitely generated submonoids is undecidable for the following two finitely generated metabelian groups: (i) the free metabelian group of rank 2 and (ii) the wreath product $\mathbb{Z} \wr (\mathbb{Z} \times \mathbb{Z})$. These results will be deduced from the following result: subsemimodule membership is undecidable for finite rank free $(\mathbb{Z} \times \mathbb{Z})$ -modules. The proof for this result involves an encoding of Turing machines via tilings. Using similar techniques, we can also show that membership in rational subsets for the two-dimensional lamplighter group $(\mathbb{Z}/2\mathbb{Z}) \wr (\mathbb{Z} \times \mathbb{Z})$ is undecidable (it remains open, whether even submonoid membership is undecidable for this group). These results nicely contrast the following theorem of Romanovskiĭ: Every finitely generated metabelian group has a decidable generalized word problem.

John Meakin: Personal and Professional Memories
Stuart Margolis
Bar-Ilan University

Surface subgroups in some CAT(0) groups
Eduardo Martínez-Pedroza
McMaster University

We consider a class of CAT(0) groups admitting a graph of groups decompositions where vertex groups are free groups and edge groups are infinite cyclic. We provide combinatorial conditions guaranteeing the existence of surface subgroups. This is joint-work, in progress, with N. Brady and M. Forester.

Mather invariants in groups of
piecewise-linear homeomorphisms
Francesco Matucci
Centre de Recerca Matemàtica

We describe the relation between two characterizations of conjugacy in groups of piecewise-linear homeomorphisms of the unit interval: the first one, discovered by M.Brin and C.Squier, defines an invariant through the iteration of a given map and the second one, developed by M.Kassabov and the author, involves the construction of actual conjugators through a similar process. Thanks to the interplay between the techniques, we produce a simplified point of view of conjugacy that allows us to easily recover centralizers and lends itself to generalization.

Braid groups and buildings
Jon McCammond
UC Santa Barbara

In this talk I will survey several results about braid groups (and their generalizations such as Artin groups) and the building-like geometric structures on which they act.

Group presentations: infinite versus finite

Alexei Miasnikov

McGill University

The classical algorithmic problems in combinatorial group theory deal with finite presentations of groups. Is it really necessary, or desirable, or just a burden? What are "good" (finite or infinite) presentations of a group? How one can find them? Is computing with infinite presentations feasible? These are the main questions I am going to touch on in the talk. In particular, I will discuss situations when infinite presentations give clear advantages over the finite ones.

Abelian sand-pile model and self-similar groups

Tatiana Nagnibeda

University of Geneva

The Abelian sand-pile model is an example of a dynamical system displaying self-organized criticality. It is widely studied in physics and in mathematics. It is initially defined on a finite graph, but the most interesting problem is to understand its asymptotic behaviour on sequences of finite graphs which tend in some sense to an infinite graph. It occurs that natural families of such graphs, of independent interest in group theory, are provided by self-similar groups. After a mathematical introduction to the Abelian sand-pile model, I will explain how new interesting asymptotics for its limit behaviour can be obtained by studying actions of self-similar groups.

Growth of iterated monodromy groups

Volodymyr Nekrashevych

Texas A&M University

We will discuss results and problems on growth of the iterated monodromy groups of polynomials and its relation with geometry of the Julia set.

Semigroup varieties for which J and D coincide

Francis J. Pastijn

Marquette University

We shall say that a semigroup variety satisfies $J = D$ if for every semigroup in the variety the Green relations J and D coincide. We find the varieties minimal for not satisfying $J = D$. There are countably many such minimal varieties and each of these is necessarily overcommutative and generated by a single semigroup not satisfying $J = D$. It turns out that each such minimal variety is the overcommutative variety generated by a finite semigroup (that is, the smallest overcommutative variety containing this finite semigroup). This result yields a classification of the semigroup varieties satisfying $J = D$.

Twisting out fully irreducible automorphisms

Alexandra Pettet

University of Michigan

By a theorem of Thurston, in the subgroup of the mapping class group generated by Dehn twists around two curves which fill, every element not conjugate to a power of one of the twists is pseudo-Anosov. We prove an analogue of this theorem for the outer automorphism group of a rank n free group. As an application, we show that every element of $GL_n(\mathbb{Z})$ is induced by a nongeometric fully irreducible automorphism. This is joint work with Matt Clay (University of Oklahoma).

Duality and equations

Jean-Eric Pin

LIAFA, CNRS and University Paris 7

Using Stone-Priestley duality, one can show that any lattice of regular languages can be defined by a set of profinite equations. This result applies in particular to classes of regular languages defined by fragments of first order logic and can be extended to infinite words and even to trees. I will present an overview of these results.

The max-min property for (relative) one-relator groups

Steve Pride

University of Glasgow

The max-min property of a word gives rise to nice properties of (relative) one-relator groups based on that word. I will discuss this property, and give some examples.

S.J. Pride, *On the residual finiteness and other properties of (relative) one-relator groups*, Proc. Amer. Math. Soc. **136** (2008) 377-386

P.H. Kropholler, S.J. Pride, K.B. Wong, P.C. Wong, *Residual finiteness of certain one-relator groups*, in preparation

A note on parabolic subgroups of a Coxeter group

Dongwen Qi

Georgia Southwestern State University

It is known that the intersection of two parabolic subgroups of a Coxeter group is a parabolic subgroup. This proposition does not imply directly that the intersection of a collection (which may contain an infinite number) of parabolic subgroups of a Coxeter group is still a parabolic subgroup. In this talk, by using the root systems and the geometric representation of a Coxeter group, we prove that the intersection of any collection of parabolic subgroups of a Coxeter group is parabolic.

The conjugacy problem as a formal language

Sarah Rees

University of Newcastle, UK

I shall discuss the language theoretic complexity of the conjugacy problem, in particular looking at the case when it is context-free, reporting on joint work with Röver and Holt. The conjugacy problems can be viewed as a set of pairs of words, and hence as a string over a finite alphabet. In fact there's more than one way to interpret the set as a string, and so I shall consider both 'synchronous' and 'asynchronous' conjugacy problems, as well as the 'inverse conjugacy problem'. We can show that finitely generated groups with asynchronously context-free inverse conjugacy problem are virtually free groups (and conversely), while the groups for which the synchronous or asynchronous conjugacy problem, or synchronous inverse conjugacy problem is context-free are precisely the virtually cyclic groups. We can also prove that for a δ -hyperbolic group the intersection of the inverse conjugacy problem with the set of pairs of quasi-geodesics is context-free.

An effective lower bound for group complexity of finite semigroups and automata

John Rhodes

University of California at Berkeley

A report on joint work with Karsten Henckell and Benjamin Steinberg, submitted for publication.

This paper presents a new lower bound for complexity that improves on all existing bounds in the literature. Using the Presentation Lemma in flow form (which will be explained), we proceed to give lower bounds by introducing a certain set/partition lattice L , and a certain set M of closure operators on $L \times L$.

These latter M have a new monoid multiplication defined on them (to be explained). We use the monoid M to facilitate the lower bounds by applying Type I-Type II bounds in conjunction with the decidability of stable pairs with respect to aperiodics (recently proved by the authors). Then by evaluating members of M at the base point, we obtain “inevitable” sets and partitions, yielding a lower bound.

Algorithmic problems in amalgams of finite inverse semigroups

Emanuele Rodaro

Università dell’Insubria

The word problem for the free product with amalgamation is undecidable (M.Sapir), however a recent paper of Cherubini, Meakin, Piochi has shown that the word problem in the amalgamated free product of two finite inverse semigroups $S_1 *_U S_2$ is decidable. We shows that there are other problems which are algorithmically solvable in $S_1 *_U S_2$, in particular we consider the solvability of equations and the problem of checking wether $S_1 *_U S_2$ has a bicyclic monoid. We consider amalgams of finite inverse semigroups where the words problem was proved to be decidable and in this framework we discuss some algorithmic questions.

The twisted conjugacy problem in a class of solvable groups

V. Romankov

Università dell’Insubria

It is proved that the twisted conjugacy problem is decidable for any endomorphism identical modulo the derived subgroup M' in any finitely generated metabelian group M , as well as for every endomorphism in any polycyclic group P . It is shown that any free nilpotent group N_{rc} of arbitrary finite rank $r \geq 2$ and class c big enough is in the Reidemeister class R_∞ .

Some regular equations in group amalgams
Gerhard Rosenberger
Universit at Dortmund

We discuss regular equations $w(x,y,z) = 1$ in free products with amalgamation and HNN-groups, especially the structure of the subgroup generated by x,y and z .

Growth of generating sets of direct powers
Nik Ruškuc
University of St Andrews

For an algebraic structure A denote by $d(A)$ the smallest size of a generating set for A , and let $d(A) = (d(A), d(A^2), d(A^3), \dots)$ (direct powers of A). Thus, for example, for a cyclic group of order n we have $d(C_n) = (1, 2, 3, 4, \dots)$. Wiegold, in a sequence of papers stretching from 1974 to 1989, and involving several co-authors, investigated the d -sequence for finite groups, infinite groups and finite semigroups. As a *very* rough summary, they prove that the d -sequence of a (non-trivial) finite group grows either linearly or logarithmically; for infinite groups constant sequences are also possible, while for finite semigroups we can have exponential growth.

In my talk I am going to report on joint work in progress with Martyn Quick and several other co-authors. In particular: (a) ‘Wiegold-type’ classifications of growth rates for rings, (classical) algebras, modules and Lie algebras; (b) connections with Universal Algebra via congruence permutability and functional completeness; (c) some curious examples of growth for infinite semigroups; (d) some initial observations for lattices, tournaments and Steiner triple systems.

Generalized expanders
Lucas Sabalka
Binghamton University

Tessera and Ostrovskii have independently introduced a generalized notion of expander in terms of probability measures on metric spaces. In joint work with Jerry Kaminker, we analyze certain classes of these generalized expanders. In this context we study, among other results, the obstruction to being able to uniformly embed a metric space into a Hadamard manifold.

Lacunary hyperbolic groups
Mark Sapir
Vanderbilt University

This is a joint work with A. Yu. Olshanskii and D.V. Osin. We consider a class of lacunary hyperbolic groups. Those are the groups some of whose asymptotic cones are \mathbb{R} -trees. Examples of such groups, as well as solutions of several problems about asymptotic cones and amenable groups will be presented.

Finitely generated permutative varieties

Olga Sapir
Vanderbilt University

We show that there exists an algorithm which decides whether a finite set of identities containing a permutation identity defines a variety generated by a finite semigroup or not.

On the growth of inverse semigroups

Lev Shneerson
Hunter College of CUNY

We study types of growth and connections between polynomial growth and bounded height condition in some classes of finitely generated inverse semigroups.

Sublinear time algorithms in the theory of groups and semigroups

Vladimir Shpilrain
The City College of New York

Typically, to give some information about an input, an algorithm should at least “read” the entire input, which takes linear time in “length”, or complexity, of the latter. Thus, linear time was usually considered the golden standard of achievement in computational complexity theory.

Sublinear time algorithms represent a new paradigm in computing, where an algorithm must give some sort of an answer after inspecting only a small portion of the input. Given that reading some data takes too long, it is natural to ask what properties of the data can be detected by sublinear algorithms that read only a small portion of the data. Thus, probably the most typical situation where sublinear time algorithms are considered is *property testing*.

In broad terms, property testing is the study of the following class of problems. Given the ability to perform local queries concerning a particular object (e.g., a graph, or a (semi)group element), the task is to determine whether or not the object has a specific property. The task should be performed by inspecting only a small (possibly randomly selected) part of the whole object.

One of the problems that we consider is the word problem. It is fairly obvious that testing sublinear-length subwords of a given word g cannot help in deciding whether or not $g = 1$ in G if G is a group. However, with semigroups the situation is different, and there might be (natural) examples of semigroups where the word problem admits a sublinear time solution for “most” inputs. One potential source of such examples is “positive monoids” associated with groups, i.e., monoids generated by group generators, but not their inverses. We investigate positive monoids associated with several interesting groups, including braid groups.

A tame 1-combing for Thompson's group F .

Melanie Stein

Trinity College

Almost convexity of a group with finite generating set is equivalent to the existence of a tame 1-combing with tameness function $\rho(n) = n$. Thompson's group F is not almost convex, so one could not hope for such a combing. We describe the next best thing: a tame 1-combing for F , with respect to the standard two generators, which satisfies a linear radial tameness function. Since F is not even minimally almost convex, this provides an example showing that even a relatively strong tameness condition does not imply the weakest of convexity conditions. (Joint work with Sean Cleary, Susan Hermiller, and Jennifer Taback.)

Symbolic dynamics, profinite groups and profinite monoids

Benjamin Steinberg

Carleton University

There is a surprising link, discovered by Jorge Almeida, between symbolic dynamics and profinite groups. The intermediary between these two fields is the free profinite monoid. A (profinite) maximal subgroup of the free profinite monoid can be associated to each irreducible symbolic dynamical system and is a conjugacy invariant of the system. With this serving as motivation, in this talk we survey some recent results about the structure of maximal subgroups of free profinite monoids, in particular those associated with minimal dynamical systems and with irreducible sofic shifts. This talk encompasses work of Jorge Almeida, Alfredo Costa, John Rhodes and the speaker.

Maximal subgroups of Inverse Semigroups

J. B. Stephen

Northern Illinois University

Let S be an inverse monoid, presented by generators X and relations T , $S = Inv \langle X | T \rangle = X^{(*)}/\tau$, where $X^{(*)}$ denotes the free monoid with involution over X . Similarly, the group with the same presentation is denoted $G = Gp \langle x | T \rangle = X^{(*)}/\sigma$, and note that G is the maximal group homomorphic image of S .

We discuss the construction of a *surface graph* of an element of S from the Schützenberger graph. The construction is reminiscent of a Riemann surface over the the Cayley graph of G .

Various applications of the representations via surface graphs are discussed: limitations of the approach; the embedding of the Schützenberger group into the wreath product of a subgroup of G and an automorphism group of the surface lattice; and, generalizable aspects of the technique to a local group dependent upon the \mathcal{J} class.

Forest algebras: an algebraic theory for automata on unranked trees

Howard Straubing

Boston College

Automata operating on trees have been studied since the 1960's, initially in connection with Mathematical Logic, and subsequently because of their application in automated verification of computer hardware and software. More recently there has been an interest, motivated by XML document processing, in automata operating on unranked trees: those in which there is no a priori bound on the number of children a node can have. Recently, M. Bojanczyk and I. Walukiewicz presented the beginnings of an algebraic theory for unranked trees by introducing *forest algebras*, and the *syntactic forest algebra* of a regular language of unranked forests. A forest algebra is little more than a pair (H, V) of monoids together with an action of V on the set H . A number of significant and nontrivial results have already been found characterizing the expressive power of logics on trees in terms of the syntactic forest algebra, and these point to the beginnings of an ideal theory and decomposition theory for these algebras. In this talk we will present the motivating problems, give the definition and basic properties of forest algebras, and survey the results discovered to date and the challenges that remain.

Closed self-similar groups of tree automorphisms

Zoran Šunić
Texas A&M

It is easy to see that closed self-similar groups of tree automorphisms are precisely the groups defined by forbidden tree patterns. We show that there exists a graphical representation of all such groups using graphs with bundled edges.

Note that there exist groups defined by forbidden patterns that are not defined by finitely many forbidden patterns (this cannot happen in the standard symbolic dynamics, which is actually defined on a rooted tree consisting of a single path that is infinite in one direction).

We present applications to finitely constrained groups (groups defined by finitely many forbidden patterns) defined by patterns of small size, such as Grigorchuk group and Hanoi Towers group.

Free Limits of Thompson's group F

Jennifer Taback

Bowdoin College

We investigate limits of marked copies of Thompson's group F within the space \mathcal{G}_3 of all marked three generator groups. We present a sequence of markings S_n of F so that the sequence (F, S_n) converges to the free group on three generators. This is interesting as F itself has no free subgroups. Recently, Matt Brin has produced a sequence of markings of F in \mathcal{G}_2 which converge to the free group on two generators. While our results easily extend to $n \geq 3$, they do not currently include the case $n = 2$. In addition, we give presentations for the limits of some other natural (convergent) sequences of markings to consider on F within \mathcal{G}_3 , including $(F, \{x_0, x_1, x_n\})$ and $(F, \{x_0, x_1, x_0^n\})$.

This is joint work with Azer Akhmedov and Melanie Stein.

Existence, covolumes and infinite generation of lattices for Davis complexes

Anne Thomas

Cornell University

Let Σ be the Davis complex for a Coxeter system (W, S) . The automorphism group G of Σ is naturally a locally compact group, and a simple combinatorial condition due to Haglund–Paulin determines when G is nondiscrete. The Coxeter group W may be regarded as a uniform lattice in G . We show that many such G also admit a nonuniform lattice Γ , and an infinite family of uniform lattices with covolumes converging to that of Γ . We also show that the nonuniform lattice Γ is not finitely generated.

Notions of hyperbolicity in monoids

Rick Thomas

University of Leicester

The notion of hyperbolic groups has played a fundamental role in group theory. There were several equivalent definitions of the notion of a hyperbolic group but one can argue that none of these generalize naturally to monoids. This changed with Gilman's elegant characterization of hyperbolic groups in terms of context-free languages; Duncan and Gilman then suggested that this formulation could be taken as the definition of a hyperbolic monoid.

Their definition is entirely natural but we do not seem to have efficient algorithms for dealing with hyperbolic monoids; for example, the word problem for hyperbolic groups can be solved in linear time but the best known algorithm for hyperbolic monoids is exponential. The purpose of this talk is to explain how restricting the definition used can lead to (efficient) algorithms.

Algorithmic and combinatorial methods for computing the relators of a group presentation

Carmelo Vaccaro

University of Paris 7, Univ. of Palermo

Let $\mathcal{P} := \langle X|R \rangle$ be a group presentation; we present a family of algorithms whose outputs are exactly the relators of \mathcal{P} . Let \mathcal{P} be a finite presentation and let a and l be natural numbers; then this gives a finite algorithm for computing all the relators of area equal to a and length l .

Given a relator w and a natural number m , this algorithm can also compute in a finite time the van Kampen diagrams for w with m faces and the expressions of w as a product of m conjugates of defining relators. This has applications for instance in the problem of finding identities among relations in \mathcal{P} .

Interpreting graphs in 0-simple semigroups with reversion

Mikhail Volkov

Ural State University, Ekaterinburg, Russia

(Joint work with Marcel Jackson, La Trobe University, Australia.)

We consider the varieties of unary semigroups generated by certain ‘adjacency semigroups’, which are combinatorial Rees matrix semigroups with unary operation $(i, j) \mapsto (j, i)$. The identities of these structures precisely capture a natural notion of equivalence modulo adjacency patterns in unary semigroup words. We establish a surprisingly close relationship between universal Horn classes of graphs and varieties generated by adjacency semigroups. For example, the lattice of subvarieties of the variety generated by adjacency semigroups that are regular unary semigroups is essentially the same as the lattice of universal Horn classes of reflexive graphs. A number of examples follow, including new examples of limit (minimal non-finitely based) varieties of unary semigroups and first examples of finite unary semigroups with NP-hard pseudovariety membership problems.

Distributions of finitely generated subgroups of free groups

Pascal Weil

Université de Bordeaux, CNRS

When considering the distribution of finitely generated subgroups of a finite rank free group F , or the problem of randomly generating such subgroups, one may consider several approaches.

One may fix a number k of generators, and randomly generate a k -tuple of reduced words in F . This was considered by several authors, e.g. Arjantseva, Myasnikov, Jutsikawa and others. Randomly generating a word of given length is a well understood process, but some work is involved to analyze the resulting distribution, since several k -tuples will produce the same subgroup.

One may instead fix the number n of vertices in the representation of the subgroup by means of a Stallings graph, and randomly generate such graphs of size n . The combinatorics of the generation algorithm is more intricate (although the algorithm itself is fast enough), but the advantage is that the discrete objects generated are in bijection with the subgroups of F . This approach was developed by Bassino, Nicaud and Weil.

The objective of the talk is to present these two approaches, including a glimpse into their combinatorial analytic justification, and the main results they have brought so far. In particular, we will compare the resulting notions of negligibility and genericity (of properties of subgroups). For instance, the property of being Whitehead minimal (to have a Stallings graph of minimal size in its automorphic orbit) is generic in both distributions. On the other hand, malnormality is generic in one distribution and negligible in the other.

Subgroup distortion in the generalized Thompson groups

Claire Wladis

BMCC/CUNY

We will describe some interesting behavior of subgroup embeddings in the generalized Thompson groups $F(n_1, \dots, n_k)$. Similar results hold for generalizations of T and V . For example, F itself is distorted in any group $F(n_1, \dots, n_k)$ where one of the $n_i = 2$, and many direct products of Thompson groups are distorted, but cyclic subgroups are quasi-isometrically embedded.

Dehn functions of linear groups

Kevin Wortman

University of Utah

I'll talk about ongoing work in determining Dehn functions of various linear groups.

Quasiisometric rigidity of certain solvable Lie groups

Xiangdong Xie
Georgia Southern University

Let $G = R^n \rtimes R$, where R acts on R^n by a diagonal matrix A that is not a multiple of the identity matrix and has all the diagonal entries larger than 1. We show that each self-quasiisometry of G is an almost isometry and is height-respecting. We prove this by showing that there is a point ∞ in the ideal boundary that is fixed by all quasisymmetric maps of the ideal boundary and each quasisymmetric map of the ideal boundary is a bilipschitz map with respect to the parabolic visual metric.

Relative limit groups of the Thompson's group F

Roland Zarzycki
University of Wroclaw

Fix a group G . Let W be any class of words over G with m variables, which are reduced and non-trivial in $\mathbb{F}_t * G$. Let $(G_n)_{n < \omega}$, $G < G_n$, be a sequence of marked groups, where $G_n := (\langle g_{n,1} \cdots g_{n,m} \rangle, (g_{n,1}, \cdots, g_{n,m}))$. We say that \mathbb{G} is a W -limit group of the sequence $(G_n)_{n < \omega}$ if the following condition is satisfied for any $w \in W$

$$\mathbb{G} \models w(\bar{g}) = 1 \iff G_n \models w(\bar{g}_n) = 1 \text{ for almost all } n$$

The case of our particular interest are the sequences of groups isomorphic to F and marked by 3 elements. However, some of our results have more general character and can be applied to other interesting groups, for example the Grigorchuk group. We classify W -limit groups for some natural classes of words.

Partial automata

Ellen Ziliak
Colorado State University

In this talk we will let G be a finite group for which we also have a presentation $\langle S | R \rangle$, (i.e. $G \cong F/N$ with $F = \langle S \rangle$ and N the normal closure of R in F). We assume that G does not have a confluent rewriting system. We want to rewrite a word in S , representing the identity in G as a product of conjugates in R . Such rewriting can be done using an (augmented) coset table for N in F which can be visualized in a graph by a coset automata. Tracing in the graph through words in F will allow us to rewrite these words as a product of generators of N . The difficulty that arises in this approach lies in storing and constructing the augmented coset table. Instead we will construct an object called a partial automata which is a subgraph of the coset automata. We will then use this object to do the same rewriting.